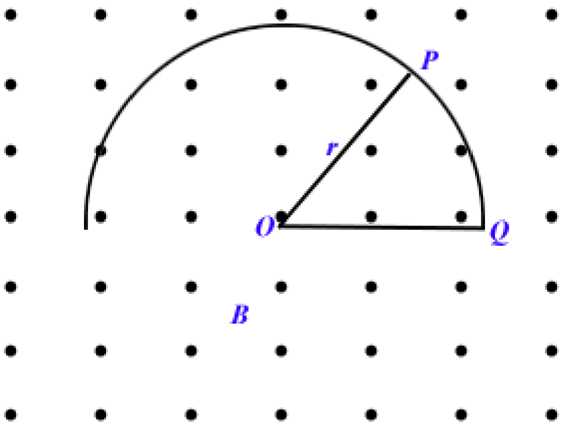
499.

Problem 36.38 (RHK)

A wire whose cross-sectional area is $1.2\ \text{mm}^2$ *and whose resistivity is* $1.7\times10^{-8}\ \Omega\ \text{m}$ *is bent into a circular arc of radius* $r = 24\ \text{cm}$ *as shown in the figure. An additional straight length of this wire, OP, is free to pivot about O and makes sliding contact with the arc at P. Finally, another straight length of this wire, OQ, completes the circuit. The entire arrangement is located in a magnetic field* $B = 0.15\ \text{T}$ *directed out of the plane of the figure. The straight wire OP starts from rest with* $\theta = 0$ *and has a constant angular acceleration of* $12\ \text{rad s}^{-2}$. (a) *We have to find the resistance of the loop OPQO as a function of* θ. (b) *We have to find the magnetic flux through the loop as a function of* θ. (c) *We have to find the value of the angle* θ *for which induced current in the loop is a maximum.* (d) *We have to find the maximum value of the induced current in the loop.*



Solution:

(a) and (b)

As the wire *OP* is moving with constant acceleration,

$\alpha = 12 \text{ rad s}^{-2}$,

and OP starts from rest with $\theta = 0$, the change with time of angle θ will be given by the function

$$\theta(t) = \frac{1}{2}\alpha t^2.$$

The flux enclosed by the loop *OQPO* will, therefore, be

$$\Phi(t) = \frac{1}{2}r^2\theta(t)B = \frac{1}{4}\alpha r^2 B t^2.$$

By the Faraday's law of induction with the change in flux in the loop emf will get developed, which is given by

$$\left|\mathrm{E}(t)\right| = \frac{d\Phi(t)}{dt} = \frac{1}{2}\alpha r^2 Bt = \frac{1}{2}\alpha r^2\left(\frac{2\theta}{\alpha}\right)^{1/2} B.$$

The resistance of the loop will change with time. The length of the arc PQ is changing because P is sliding along the circumference of the semicircle. The resistance of the arc will be given by the function

$$R(\theta) = (2r + r\theta)\frac{\rho}{a},$$

where ρ is the resistivity and a is the cross-sectional area of the wire.

(c) and (d)

Induced current in the loop as a function of θ will be given by the expression

$$i(\theta) = \frac{\mathrm{E}(\theta)}{R(\theta)} = \frac{\frac{1}{2}r^2\alpha^{\frac{1}{2}}2^{\frac{1}{2}}\theta^{\frac{1}{2}}B}{r(2+\theta)\rho/a} = \frac{1}{\sqrt{2}}\frac{r\alpha^{\frac{1}{2}}\theta^{\frac{1}{2}}B}{(2+\theta)\rho/a}.$$

For finding the maximum value of $i(\theta)$ we will calculate the extremum of the function $i(\theta)$.

$$\frac{di(\theta)}{d\theta} = 0,$$

or

$$\frac{1}{2}\frac{\theta^{-\frac{1}{2}}}{(2+\theta)} + \frac{\theta^{\frac{1}{2}}(-1)}{(2+\theta)^2} = 0,$$

or

$$\frac{1}{2}(2+\theta) - \theta = 0,$$

or

i will be maximum for $\theta = 2$ rad.

The maximum value of the induced current will therefore be

$$i(\theta = 2\text{ rad}) = \frac{1}{\sqrt{2}}\frac{r\alpha^{\frac{1}{2}}\theta^{\frac{1}{2}}B}{(2+\theta)\rho/a}$$

$$= \frac{1}{\sqrt{2}}\frac{0.24\times(12)^{\frac{1}{2}}\times\sqrt{2}\times0.15\times1.2\times10^{-6}}{(2+2)\times1.7\times10^{-8}}\text{ A}$$

$$= 2.2\text{ A}.$$