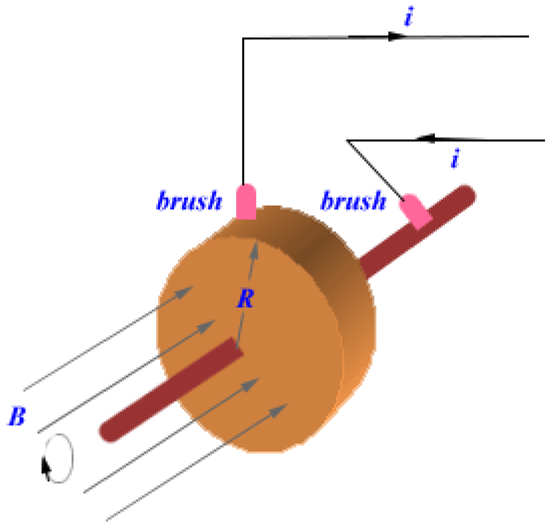
498.

Problem 36.36 (RHK)

In the figure a device called "homopolar generator" consisting of a solid conducting disk as rotor has been shown. This machine can produce a greater emf than one using wire loop rotors, since it can spin at a much higher angular speed before centrifugal forces disrupt the rotor. (a) *We have to show that the emf produced is given by*

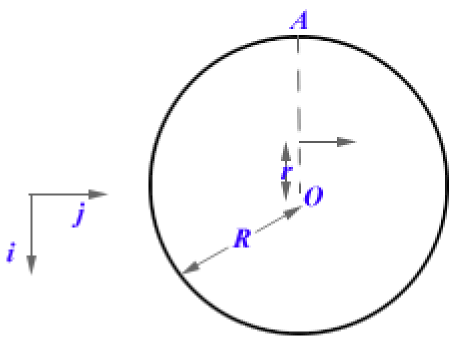
$$\mathrm{E} = \pi \nu B R^2,$$

where ν *is the spin frequency, R the rotor radius, and B the uniform magnetic field perpendicular to the rotor.* (b) *We have to find the torque that must be provided by the motor spinning the rotor when the output current is* i.



Solution:

As the conducting disk is rotating with angular velocity $2\pi\nu$, the free charge carriers inside the disk will rotate with the disk and with respect to the laboratory observer acquire tangential velocity which will depend linearly on the distance of the charge from the centre of the disk. Therefore, the speed of a charge carrier at a distance r from the centre of the disk will be $2\pi\nu r$.



Because of the magnetic field $\vec{B}$, which is normal to the velocity of the carrier, the charge carrier (charge e) will experience Lorentz force of magnitude $2\pi\nu reB$.

If at an instant we consider a charge carrier located at a distance *r* along the line *OA*, its velocity will be $\vec{v} = 2\pi r\nu\hat{j}$. As shown in the diagram magnetic field on the disk is $\vec{B} = B\hat{k}$.

Therefore,

$\vec{v} \times \vec{B} = 2\pi r\nu B\hat{i}$.

Therefore, if the charge carriers are electrons having charge $-e$, the Lorentz force will be along the direction $-\hat{i}$.

Electrons will drift toward the circumference of the disk and an electric field will come into being (like in Hall effect) that at equilibrium will balance the force $2\pi r\nu B\hat{i}$. The electric field will therefore be

$\vec{E}(r) = -2\pi r\nu B\hat{i}$.

The potential difference between the points *A* and *O* will therefore be

$$V_A - V_O = -\int_O^A \vec{E}.d\vec{r} = -2\pi\nu B\int_0^R r dr = -\pi\nu BR^2 = -E.$$

If the output current is i, the power dissipated in the rotor will be

$$P = (V_O - V_A)i.$$

Therefore, a torque, τ, must be provided by the motor for keeping the rotor spinning at frequency ν. The magnitude of τ can now be determined by equating the power of the motor with the rate of dissipation of internal energy in the rotor. That is

$$\tau\omega = \pi\nu BR^2 i,$$

or

$$\tau = \frac{BR^2 i}{2}.$$