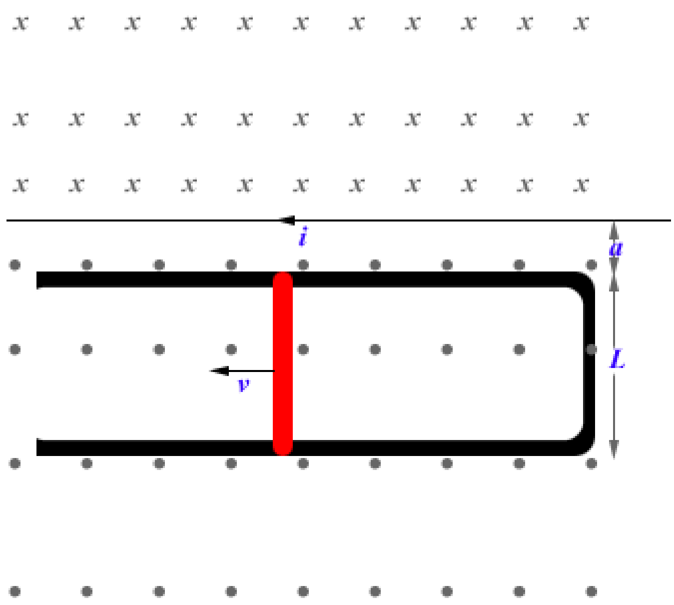
497.

Problem 36.33 (RHK)

A rod of length L is caused to move at constant speed v along horizontal conducting rails, as shown in the figure. In this case the magnetic field in which the rod moves is not uniform but is provided by a current i in a long parallel wire. We may assume that $v = 4.86 \text{ m s}^{-1}$, $a = 10.2 \text{ mm}$, $L = 9.83 \text{ cm}$, *and* $i = 110 \text{ A}$. *We have to calculate* (a) *the emf induced in the rod; and* (b) *find the current in the conducting loop. We may assume that the resistance of the rod is* $415 \text{ m}\Omega$ *and that the resistance of the rails is negligible.* (c) *We have to calculate the rate at which internal energy is being generated in the rod.* (d) *We have to calculate the force that must be applied to the rod by an external agent to maintain its motion.* (e) *We have to find the rate at which the external agent does work on the rod and compare our answer with that found in* (c).



Solution:

From Ampere's law and considering that the magnetic field due to a long current carrying wire will be circular, we note that the magnitude of the field at a distance *x* from the wire will be

$$B = \frac{\mu_0 i}{2\pi x}.$$

The flux enclosed in the loop formed by the rod and the conducting rails when the rod is at distance *y* from the left-hand end will be given by the integral

$$\Phi = \int_{a}^{a+L} \frac{\mu_0 i}{2\pi x} y dx = \frac{\mu_0 i y}{2\pi} \ln\left(\frac{a+L}{a}\right).$$

Flux enclosed is changing with time as the conducting rod is being made to move with constant speed

$v = dy/dt$. By the Faraday's law the induced emf in the circuit will be given by

$$|\mathrm{E}| = \frac{d\Phi}{dt} = \frac{\mu_0 i v}{2\pi} \ln\left(\frac{a+L}{a}\right).$$

Data for the problem are;

$v = 4.86 \text{ m s}^{-1}$,

$a = 10.2 \text{ mm}$,

$L = 9.83 \text{ cm}$,

and

$i = 110 \text{ A}$.

(a)

The induced emf developed in the loop will be

$$\mathrm{E} = 2\times10^{-7} \times 110 \times 4.86 \times \ln\left(\frac{9.83+1.02}{1.02}\right) \text{ V}$$

$$= 2.528\times10^{-4} \text{ V} = 252.8 \ \mu\text{V}.$$

(b)

As the resistance in the circuit formed by the conducting rod is $415 \text{ m}\Omega$, the induced current in the loop will be given by

$$i_{induced} = \frac{\mathrm{E}}{R} = \frac{252.8\times10^{-6}}{415\times10^{-3}} \text{ A} = 0.609\times10^{-3} \text{ A} = 609 \ \mu\text{A}.$$

(c)

The rate at which internal energy is being generated in the rod will be given by

$$P = \mathrm{E} i_{induced} = 252.8\times10^{-6}\times609\times10^{-6}\ \mathrm{W} = 153.9\ \mathrm{nW}.$$

(d)

The force that must be applied by an external agent to maintain the motion of the conducting rod will be given by integrating the Lorentz force on element of length dx at a distance x from the wire carrying current $i_{induced}$ over the length of the conductor,

$$dF = i_{induction} dx \times \left(\frac{\mu_0 i}{2\pi x}\right),$$

and

$$F = i_{induction}\frac{\mu_0 i}{2\pi}\int_a^{a+L}\frac{dx}{x} = i_{induction}\frac{\mu_0 i}{2\pi}\ln\left(\frac{a+L}{L}\right)$$

$$= i_{induction}\frac{\mathrm{E}}{v} = \frac{P}{v} = \frac{153.9\times10^{-9}}{4.86}\ \mathrm{N} = 31.6\ \mathrm{nN}.$$

(e)

The rate at which the external agent does work on the rod will be given by

$$Fv = 31.6\times10^{-9}\times4.86\ \mathrm{W} = 153.9\ \mathrm{nW}.$$

It is equal to the rate at which internal energy is being generated in the rod.