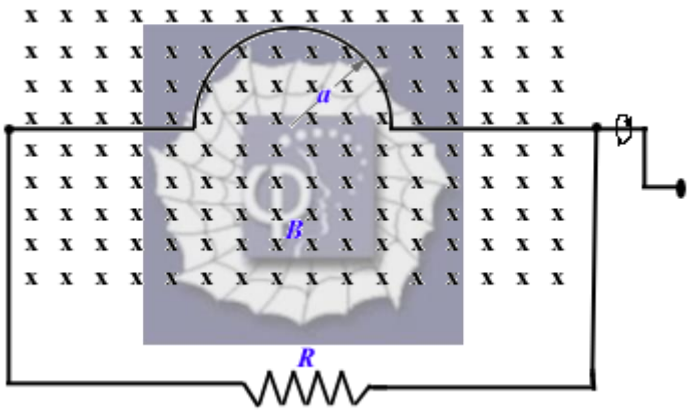
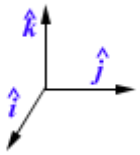
495.

Problem 36.26 (RHK)

A stiff wire bent into a semicircle of radius a is rotated with a frequency ν *in a uniform magnetic field, as suggested in the figure. We have to find* (a) *the frequency and* (b) *the amplitude of the emf induced in the loop.*





Solution:

(a) and (b)

Let us assume that at an instant t the plane of the loop makes an angle θ with respect to the vertical. The unit

vector $\hat{n}$ perpendicular to the plane of the loop at that instant will be

$$\hat{n} = \cos\theta\hat{i} - \sin\theta\hat{k}.$$

As the magnetic field in which the semicircular loop of radius a is revolving is

$$\vec{B} = -B\hat{i},$$

the flux enclosed by the loop will be

$$\Phi = \frac{\pi a^2}{2}\hat{n}.\vec{B} = \frac{\pi a^2}{2}\left(-B\sin\theta\right).$$

By Faraday's law the emf due to induction will be given by

$$\mathrm{E} = -\frac{d\Phi}{dt} = \frac{\pi a^2 B}{2}\cos\theta\frac{d\theta}{dt}.$$

As

$$\frac{d\theta}{dt} = -2\pi\nu,$$

$$\theta = -2\pi\nu t + \theta_0.$$

The induced emf will therefore be given by the expression

$$\mathrm{E} = \frac{\pi a^2 B}{2}\left(2\pi\nu\right)\cos\left(2\pi\nu t - \theta_0\right) = \pi^2 a^2 B\nu\cos\left(2\pi\nu t - \theta_0\right).$$

The frequency of the induced emf will therefore be ν and its amplitude will be $\pi^2 a^2 B\nu$.