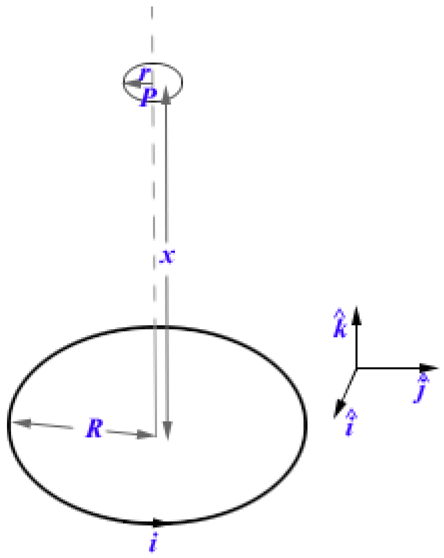
494.

<u>Problem 36.24 (RHK)</u>

In the figure two parallel loops of wire having a common axis have been shown. The smaller loop (*radius r*) *is above the larger loop* (*radius R*), *by a distance* $x ? R$. *Consequently the magnetic field, due to the current i in the larger loop, is nearly constant throughout the smaller loop and equal to the value on the axis. Suppose that x is increasing at constant rate* $dx/dt = v$. (a) *We have to determine the magnetic flux across the area bounded by the smaller loop as a function of x.* (b) *We have to compute the emf generated in the smaller loop.* (c) *We have to determine the direction of the induced current flowing in the smaller loop.*



<u>Solution:</u>

As the point P is on the axis of the loop of radius R and is at a distance x from the plane of the loop such that

x ? R,

and a current i is flowing in the counter-clockwise direction, the magnetic field at P will be approximately given by

$$\vec{B}(0,0,x) = \frac{\mu_0 i R^2}{2x^3}\hat{k}.$$

It is the magnetic field due to a dipole of moment $\pi R^2 i$ at a point on its axis far away from the dipole at a distance x from it.

In this approximation we can assume that the magnetic field in the region enclosed by the loop of radius r will be uniform and equal to the value of the field at the centre of the circle. Therefore, the flux enclosed by the loop of radius *r* will be

$$\Phi(x) = \frac{\mu_0 i R^2}{2x^3} \times \pi r^2 = \frac{\mu_0 \pi i R^2 r^2}{2x^3}.$$

As the smaller loop is moving in the direction of the axis with speed *v*, the induced emf in it will be given by the Faraday's law,

$$\mathrm{E} = -\frac{d\Phi}{dt} = -\frac{d}{dt}\left(\frac{\mu_0 i \pi R^2 r^2}{2x^3}\right) = \frac{3\mu_0 i \pi R^2 r^2}{2x^4}\frac{dx}{dt}$$

$$= \frac{3\mu_0 i \pi R^2 r^2 v}{2x^4},$$

as

$$\frac{dx}{dt} = v.$$

Therefore, the induced emf in the loop of radius *r* will be

$$\mathrm{E} = \frac{3\mu_0 i \pi R^2 r^2 v}{2x^4}.$$

As the flux enclosed in the loop of radius *r* will decrease as it moves up along the axis, therefore, by the Lenz' law the direction of the induced current will be counter-clockwise as seen from the top along the line joining the centres of the two loops.