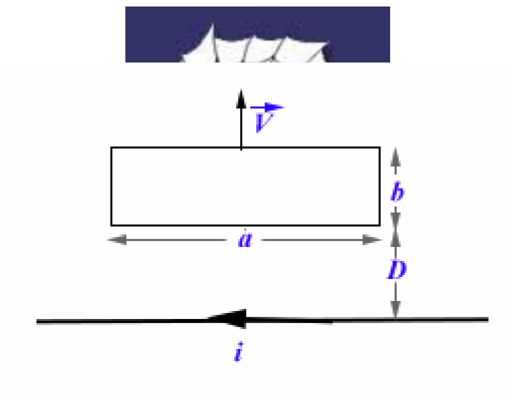
490.

<u>Problem 36.35 (RHK)</u>

A rectangular loop of wire of with length a, width b, and resistance R is placed near an infinitely long wire carrying current i, *as shown in the figure. The distance from the long wire to the loop is D. We have to find* (a) *the magnitude of the flux through the loop and* (b) *the current in the loop as it moves away from the long wire with speed v.*



<u>Solution:</u>

By applying the Ampere's law, we note that the magnetic field at a distance ξ from the infinitely long wire carrying current i will be given by

$$B(\xi) = \frac{\mu_0 i}{2\pi\xi}.$$

From the right-hand rule and looking at the direction of the current in the infinitely long wire, we note that the magnetic lines of force will be entering from the top of the page into it. The magnitude of the magnetic flux through the loop will be given by the following integral:

$$\Phi = \int_{D}^{D+b} a\, d\xi B(\xi) = \frac{a\mu_0 i}{2\pi} \int_{D}^{D+b} \frac{d\xi}{\xi} = \frac{a\mu_0 i}{2\pi} \ln\left(\frac{D+b}{D}\right).$$

It is given that the loop is moving away from the long wire with speed *V*. The distance of the loop from the wire will, therefore, be given by the function

$$D(t) = D(0) + Vt.$$

Substituting $D(t)$ in the expression for Φ, we get

$$\Phi(t) = \frac{a\mu_0 i}{2\pi} \ln\left(\frac{D(0)+Vt+b}{D(0)+Vt}\right).$$

$$\therefore \quad \frac{d\Phi(t)}{dt} = \frac{a\mu_0 i}{2\pi}\left(\frac{V}{D(0)+Vt+b} - \frac{V}{D(0)+Vt}\right)$$

$$= -\frac{ab\mu_0 iV}{2\pi D(D+b)}.$$

By Faraday’s law, the induced emf in the circuit will therefore be given by

$$\mathrm{E} = -\frac{d\Phi}{dt} = \frac{abiV\mu_0}{2\pi D(D+b)}.$$

It is given that the resistance of the loop is *R*. Therefore, by Ohm’s law the current in the loop will be

$$i = \frac{\mathrm{E}}{R} = \frac{abiV\mu_0}{2\pi RD\left(D+b\right)}.$$