488.

Problem 36.16 (RHK)

In the circuit shown in the figure, a = 12 cm and b = 16 cm. The current in the long wire is given by $i = 4.5t^2 - 10t$, where i is in amperes and t is in seconds. We have to find the emf in the square loop at t = 3.0 s.



Solution:

We note that the magnetic field due to current in a long wire will be circular. As the current is flowing from left to right (west to east) the magnetic lines of force will emerge out of the plane of the page in the upper half side of the wire and will be entering in the plane of the page in the lower half of the plane of the page.

The magnitude of the magnetic field will depend on distance from the wire. Applying Ampere's law, we have

$$2\pi\xi B(\xi) = \mu_0 i,$$

or

$$B(\xi) = \frac{\mu_0 i}{2\pi\xi}.$$

The flux of the magnetic field entering the plane of the square loop will be given by the integral

$$\Phi_{(+)} = \int_{\varepsilon}^{a} \frac{b\mu_{0}id\xi}{2\pi\xi} = \frac{b\mu_{0}i}{2\pi} \ln\left(\frac{a}{\varepsilon}\right),$$

where ε is the radius of the wire carrying the current. Similarly, the flux of the magnetic field coming out of the plane of the page will be given by the integral

$$\Phi_{(-)} = \int_{\varepsilon}^{b-a} \frac{b\mu_0 id\xi}{2\pi\xi} = \frac{b\mu_0 i}{2\pi} \ln \frac{b-a}{\varepsilon}$$

The net flux entering the square loop will be

$$\Phi = \Phi_{(+)} - \Phi_{(-)} = \frac{b\mu_0 i}{2\pi} \ln\left(\frac{a}{b-a}\right).$$

Data of the problem are:

 $a = 12 \text{ cm}, b = 16 \text{ cm} \text{ and } i = 4.5t^2 - 10t.$

$$\Phi = 16 \times 10^{-2} \times 2 \times 10^{-7} \times \left(4.5t^2 - 10t\right) \ln\left(\frac{12}{4}\right) \mathrm{T m}^2,$$

and

$$\frac{d\Phi}{dt} = 32 \times 10^{-9} \times (9.0t - 10.0) \ln 3 \text{ T m}^2 \text{ s}^{-1}.$$

The emf in the square loop at t = 3.0 s will, therefore, be given by

$$\frac{d\Phi(t=3.0)}{dt} = 32 \times 10^{-9} \times (27-10) \ln 3 \text{ V} = 0.597 \ \mu\text{V}.$$

As the flux entering the loop is increasing with time, by the Lenz' law we note that the direction of the induced current in the loop will be counter-clockwise.

