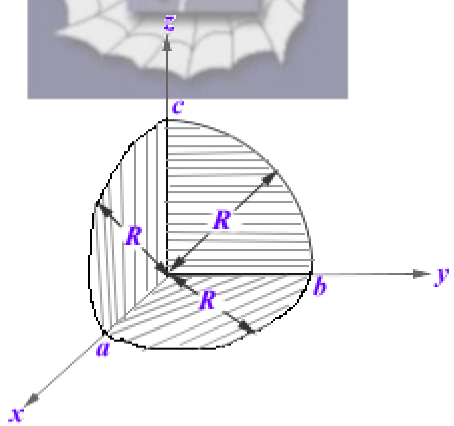
487.

Problem 36.15 (RHK)

A wire is bent into three circular segments of radius $r = 10.4$ cm, *as shown in the figure. Each segment is a quadrant of a circle, ab lying in the xy plane, bc lying in the yz plane, and ca lying in the zx plane.* (a) *If a uniform magnetic field* $\vec{B}$ *points in the positive x direction, we have to find the emf developed in the wire when* $\vec{B}$ *increases at the rate of* 3.32 mT s^{-1}. (b) *We have to find the direction of the emf in the segment bc.*



Solution:

A wire is bent into three circular segments of radius 10.4 cm. Each segment is a quadrant of a circle, *ab* lying in the *xy* plane, *bc* lying in the *yz* plane, and *ca* lying in the *zx* plane. A uniform magnetic field $\vec{B}$ points in the positive *x* direction.

$\vec{B} = B\hat{i}$.

We have to find the emf developed in the wire when B increases at the rate of $3.32\ \text{mT s}^{-1}$.

For calculating the flux through the surface bounded by the segments *ab*, *bc* and *ca*, we will use spherical polar coordinates.

$z = R\cos\theta,$

$x = R\sin\theta\cos\varphi,$

$y = R\sin\theta\sin\varphi.$

The unit vector $\hat{R}$ normal to the spherical surface is

$$\hat{R} = \frac{x}{R}\hat{i} + \frac{y}{R}\hat{j} + \frac{z}{R}\hat{k}.$$

Therefore, the flux through the surface indicated as shown in the figure will be given by the integral

$$\Phi = \int_0^{\pi/2} d\varphi \int_0^{\pi/2} R^2 \sin\theta d\theta \left(\hat{R}.B\hat{i}\right)$$

$$= \int_0^{\pi/2} d\varphi \int_0^{\pi/2} R^2 \sin\theta d\theta B \frac{x}{R}$$

$$= \int_0^{\pi/2} d\varphi \int_0^{\pi/2} R^2 \sin\theta d\theta B \sin\theta \cos\varphi$$

$$= R^2 B \int_0^{\pi/2} \sin^2\theta d\theta = \frac{R^2 B\pi}{4}.$$

By Faraday's law of induction,

$$\mathrm{E} = -\frac{d\Phi}{dt} = -\frac{\pi R^2}{4}\frac{dB}{dt}.$$

It is given that

$R = 10.4\times10^{-2}$ m,

and

$$\frac{dB}{dt} = 3.32\times10^{-3}\ \mathrm{T\ s^{-1}}.$$

$$\therefore\ \mathrm{E} = \frac{\pi\times\left(10.4\times10^{-2}\right)^2\times3.32\times10^{-3}}{4}\ \mathrm{V} = 28.2\ \mu\mathrm{V}.$$

We have to find the direction of the emf in the segment *bc*. We set up a circuit by joining the loop bc to the origin of the coordinate system; join *b* to the origin and from origin to *c*. As the magnetic field is in the positive *x* direction and is increasing with time, by the Lenz' law induced current will flow from *c* to *b*, i.e. in the clockwise direction to resist the increase in flux with time.