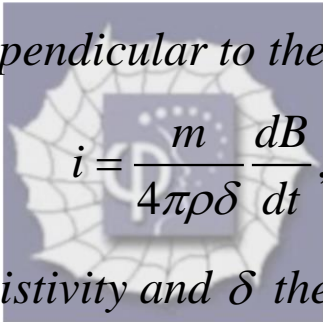


485.

**Problem 36.13 (RHK)**

A uniform magnetic field  $\dot{B}$  is changing in magnitude at a constant rate  $dB/dt$ . We are given a mass  $m$  of copper which is to be drawn into a wire of radius  $r$  and formed into a circular loop of radius  $R$ . We have to show that the induced current in the loop does not depend on the size of the wire or of the loop and, assuming  $\dot{B}$  is perpendicular to the loop is given by


$$i = \frac{m}{4\pi\rho\delta} \frac{dB}{dt},$$

where  $\rho$  is the resistivity and  $\delta$  the density of the copper.

**Solution:**

As the radius of the copper wire is  $r$  and its length is  $2\pi R$ , the mass of the wire

$$m = 2\pi R(\pi r^2)\delta,$$

where  $\delta$  is the density of copper and  $m$  is the mass of the copper from which the wire has been drawn.

Resistance of this copper wire will be

$$R_{res} = \frac{2\pi R\rho}{\pi r^2} = \frac{2R\rho}{r^2},$$

where  $\rho$  is the resistivity of the copper.

Magnitude of the induced emf in the loop due to changing magnetic field,  $dB/dt$ , perpendicular to the plane of the loop is given by the Faraday's law,

$$|E| = \pi R^2 \frac{dB}{dt}.$$

But

$$E = iR_{res} = \frac{2i\rho R}{r^2},$$

$$\therefore \pi R^2 \frac{dB}{dt} = \frac{2i\rho R}{r^2},$$

and

$$i = \frac{\pi Rr^2}{2\rho} \frac{dB}{dt}.$$

We recall that

$$\pi R^2 r = \frac{m}{2\pi\delta},$$

$$\therefore i = \frac{m}{4\pi\rho\delta} \frac{dB}{dt},$$

which is independent of the size of the wire or that of the loop.

