485.

Problem 36.13 (RHK)

A uniform magnetic field $\vec{B}$ is changing in magnitude at a constant rate dB/dt. We are given a mass m of copper which is to be drawn into a wire of radius r and formed into a circular loop of radius R. We have to show that the induced current in the loop does not depend on the size of the wire or of the loop and, assuming $\vec{B}$ is perpendicular to the loop is given by

$$i = \frac{m}{4\pi\rho\delta}\frac{dB}{dt},$$

where ρ is the resistivity and δ the density of the copper.

Solution:

As the radius of the copper wire is r and its length is $2\pi R,$ the mass of the wire

$$m = 2\pi R\left(\pi r^2\right)\delta,$$

where δ is the density of copper and m is the mass of the copper from which the wire has been drawn.

Resistance of this copper wire will be

$$R_{res} = \frac{2\pi R\rho}{\pi r^2} = \frac{2R\rho}{r^2},$$

where ρ is the resistivity of the copper.

Magnitude of the induced emf in the loop due to changing magnetic field, dB/dt, perpendicular to the plane of the loop is given by the Faraday's law,

$$|\mathrm{E}| = \pi R^2 \frac{dB}{dt}.$$

But

$$\mathrm{E} = iR_{res} = \frac{2i\rho R}{r^2},$$

$$\therefore\ \pi R^2 \frac{dB}{dt} = \frac{2i\rho R}{r^2},$$

and

$$i = \frac{\pi R r^2}{2\rho}\frac{dB}{dt}.$$

We recall that

$$\pi R^2 r = \frac{m}{2\pi\delta},$$

$$\therefore\ i = \frac{m}{4\pi\rho\delta}\frac{dB}{dt},$$

which is independent of the size of the wire or that of the loop.