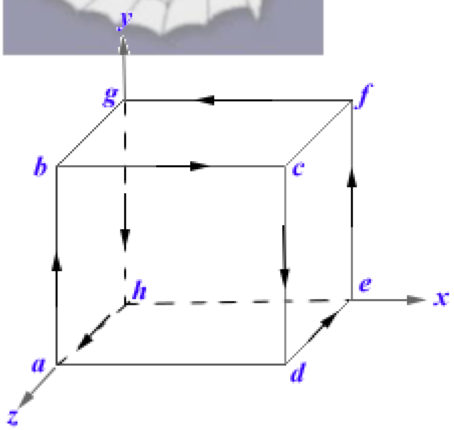
483.

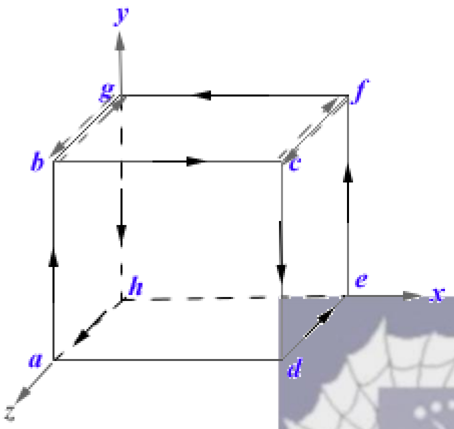
Problem 30.71 P (HRW)

A conductor carries a current of 6.0 A *along the closed path abcdefgha involving* 8 *of the* 12 *edges of a cube of side* 10 cm *as shown in the figure. We have to answer* (a) *why one can regard this as the superposition of three square loops: bcfgb, abgha, and cdefc?* (b) *By using the superposition we have to find the magnetic dipole moment* $\vec{\mu}$ *of the closed path.* (c) *We have to calculate* $\vec{B}$ *at the points* $(x, y, z) = (0, 5.0\text{ m}, 0)$ *, and* $(5.0\text{ m}, 0, 0)$.



Solution:

A conductor carries a current of 6.0 A along the closed path *abcdefgha* involving 8 of the 12 edges of a cube of side 10 cm as shown in the figure. We imagine that on the edge *bg* a current of 6.0 A is flowing from *b* to *g* , and a current of 6.0 A is flowing from *g* to *b*. similarly, we imagine that on the edge *cf* a current of 6.0 A is flowing from *c* to *f* and a current of 6.0 a is flowing in the reverse direction from *f* to *c*.



The closed current loop *abcdefgha* becomes equivalent to three square current loops *bcfgb*, *abgha*, and *cdefc*.

We replace each closed current loop by its equivalent magnetic dipole moment and determine the magnetic dipole moment of the loop *abcdefgha* as the superposition of the dipole moments for the loops *bcfgb*, *abgha*, and *cdefc*.

As the length of each edge of the cube is 0.1 m and a current of 6.0 A is flowing in current loops, the equivalent magnetic dipole moments are:

$$\vec{\mu}_{bcfgb} = 0.1^2 \times 6.0\,\hat{j}\ \text{A m}^2,$$

$$\vec{\mu}_{abgha} = -0.1^2 \times 6.0\,\hat{i}\ \text{A m}^2,$$

and

$$\vec{\mu}_{cdefc} = 0.1^2 \times 6.0\,\hat{i}\ \text{A m}^2.$$

The equivalent magnetic dipole moment of the loop *abcdefgha* will therefore be given by the superposition of the three dipole moments and will be

$$\vec{\mu}_{abcdefgha} = 0.06\hat{j} \text{ A m}^2.$$

The magnetic field at the point $(0, 5.0 \text{ m}, 0)$, as it is on the axis of the dipole and is at a distance d far compared to the dimension of the dipole, will be given by the expression

$$\vec{B} = \frac{\mu_0 \vec{\mu}}{2\pi d^3} = \frac{2\times10^{-7}\times6.0\times10^{-2}}{5^3}\hat{j} \text{ T} = 9.6\times10^{-11}\hat{j} \text{ T}.$$

The magnetic field at the point $(5.0 \text{ m}, 0, 0)$, as it is on the $\perp$ bisector to the dipole and is at a distance d far from the dimension of the dipole, will be given by the expression

$$\vec{B} = -\frac{\mu_0 \vec{\mu}}{4\pi d^3} = -4.8\times10^{-11}\hat{j} \text{ T}.$$