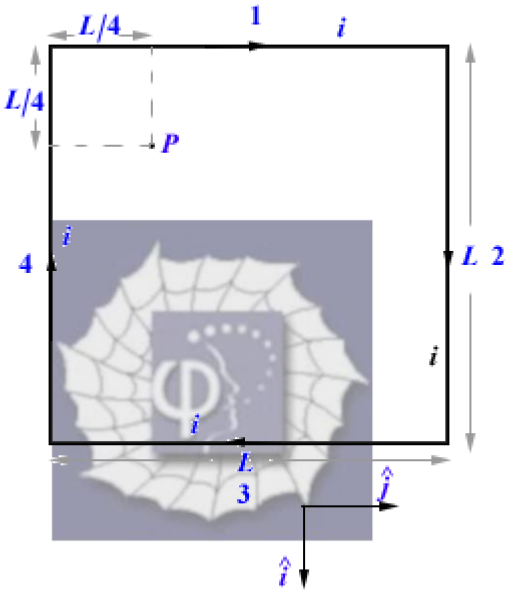
481.

<u>Problem 35.31 (RHK)</u>

We have to calculate (a) $\vec{B}$ *at point P in the figure.* (b) *We have to answer whether the field strength at P is greater or less than at the centre of the square?*

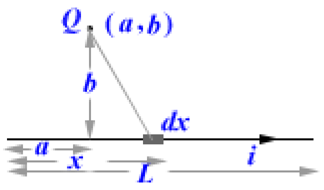


<u>Solution:</u>

A current i is flowing in the clockwise direction in a square loop of side L. We have to calculate the magnetic field at the point P, shown in the figure.

(a)

In order to answer this problem, we will first calculate the magnitude of the magnetic field due to a wire line-segment of a length L carrying a current i. In the following a line diagram has been drawn for solving this part of the problem.



The co-ordinates of the point Q are (a, b). Its vertical distance from the line segment is b. We use the Biot-Savart law for calculating the magnetic field at Q. We note that as the line segment and the point Q are in the same plane, the direction of the magnetic field will be perpendicular to the plane. Whether it is an outward normal or it is an inward normal to the plane will be determined by the direction of the current in the line segment.

The magnitude of the magnetic field at Q due to the current element idx will be

$$dB(Q) = \frac{\mu_0}{4\pi}\frac{idx}{\left(b^2+(x-a)^2\right)} \times \frac{b}{\left(b^2+(x-a)^2\right)^{1/2}}$$

$$= \frac{\mu_0 bidx}{4\pi\left(b^2+(x-a)^2\right)^{3/2}}.$$

Therefore, the magnetic field at Q due to the current line-segment of length L will be given by

$$B(Q)=\int_0^L \frac{\mu_0 bidx}{4\pi\left(b^2+(x-a)^2\right)^{3/2}}.$$

For calculating this integral, we make the substitution

$x-a=b\tan\theta,$

$dx=b\sec^2\theta.$

We have

$$B(Q)=\int_{-\tan^{-1}\left(\frac{a}{b}\right)}^{\tan^{-1}\left(\frac{L-a}{b}\right)} \frac{\mu_0 ib^2\sec^2\theta d\theta}{4\pi b^3\sec^3\theta}=\frac{\mu_0 i}{4\pi b}\int_{-\tan^{-1}\left(\frac{a}{b}\right)}^{\tan^{-1}\left(\frac{L-a}{b}\right)} \cos\theta d\theta,$$

Or

$$B(Q)=\frac{\mu_0 i}{4\pi b}\left(\sin\left(\tan^{-1}\frac{(L-a)}{b}\right)+\sin\left(\tan^{-1}\left(\frac{a}{b}\right)\right)\right)$$

$$=\frac{\mu_0 i}{4\pi b}\left(\frac{L-a}{\left((L-a)^2+b^2\right)^{1/2}}+\frac{a}{\left(a^2+b^2\right)^{1/2}}\right).$$

We will use this result for finding the contribution to the magnetic field at P due to the current line-segments 1,2,3 and 4 of the square-loop in which a current of magnitude i is flowing in the clockwise direction. From the direction of the current circulation in the loop, we note that the direction of the magnetic field will be along the inward normal to the page, that is along $-\hat{k}$.

Side 1

Coordinates of the point P with respect to the line-segment 1 are

$$a = \frac{L}{4},\ b = \frac{L}{4}.$$

$$\therefore\ \vec{B}_1(P) = \frac{\mu_0 i}{\pi L}\left(-\hat{k}\right)\left(\frac{1}{\sqrt{2}} + \frac{3}{\sqrt{10}}\right).$$

Side 2

Coordinates of the point P with respect to the line-segment 2 are

$$a = \frac{L}{4},\ b = \frac{3L}{4}.$$

$$\therefore\ \vec{B}_2(P) = \frac{\mu_0 i}{3\pi L}\left(-\hat{k}\right)\left(\frac{1}{\sqrt{10}} + \frac{1}{\sqrt{2}}\right).$$

Side 3

Coordinates of the point P with respect to the line-segment 3 are

$$a = \frac{3L}{4},\ b = \frac{3L}{4}.$$

$$\therefore\ \vec{B}_3(P) = \frac{\mu_0 i}{3\pi L}\left(-\hat{k}\right)\left(\frac{1}{\sqrt{10}} + \frac{1}{\sqrt{2}}\right).$$

Side 3

Coordinates of the point P with respect to the line-segment 4 are

$$a = \frac{3L}{4},\ b = \frac{L}{4}.$$

$$\therefore\ \vec{B}_4(P) = \frac{\mu_0 i}{\pi L}\left(-\hat{k}\right)\left(\frac{1}{\sqrt{2}} + \frac{3}{\sqrt{10}}\right).$$

Therefore, the magnetic field at P due to the current square-loop will be

$$\vec{B}(P) = \vec{B}_1(P) + \vec{B}_2(P) + \vec{B}_3(P) + \vec{B}_4(P)$$

$$= \frac{2\mu_0 i}{3\pi L}\left(-\hat{k}\right)\left(2\sqrt{2} + \sqrt{10}\right).$$

(b)

We next calculate the magnetic field at the centre of the square-loop.

As the position of the centre of the square with respect to all the four sides is the same, they will make equal contributions to the magnetic field. The contribution to the magnetic field at the centre from each of the sides of the square-loop can be obtained from the general formula by using the parameters $a = \frac{L}{4},\ b = \frac{L}{4}.$

We find

$$\vec{B}(centre) = \frac{2\sqrt{2}\mu_0 i}{\pi L}\left(-\hat{k}\right).$$

We will now compare the magnetic field strengths at the point P and that at the centre of the square-loop.

As

$$\sqrt{2}\left(\frac{2}{3}+\frac{\sqrt{5}}{3}\right) > \sqrt{2} \ ,$$

The field strength of the magnetic field at P is greater than that at the centre of the square.