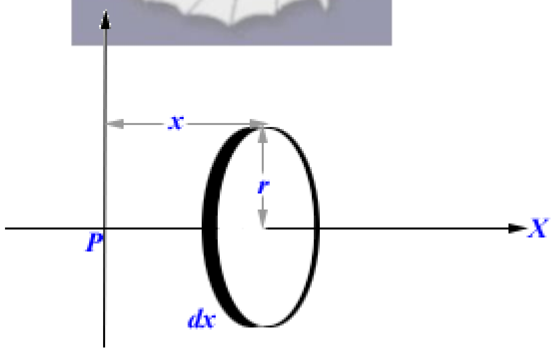
480.

<u>Problem 35.56 (RHK)</u>

We have to derive the solenoid equation starting from the expression for the field on the axis of a circular loop.

<u>Solution:</u>

Let the number of turns per unit length of the solenoid be n. We will calculate the magnetic field at the axis of an ideal solenoid. That is we assume that the solenoid is of infinite length. Let the radius of the solenoid be r.



We recall that the magnetic field at the axis of a circular current carrying loop at a distance x from its centre is given by

$$B(x) = \frac{\mu_0 i r^2}{2\left(r^2 + x^2\right)^{3/2}}.$$

Its direction is along the axis perpendicular to the plane of the circular loop.

We will calculate the magnetic field at P, which we take to be the origin of the *x*-axis. Let us consider a section of the solenoid of thickness dx at a distance x from the point P. Let the current flowing in the circular loops of the solenoid be

$$dB(P)=\frac{\mu_0 ndxir^2}{2\left(r^2+x^2\right)^{3/2}}.$$

Therefore, the magnetic field at P at the axis of the solenoid will be given by integrating $dB(P)$ with respect to x from $-\infty$ to $+\infty$. We have

$$B(P)=\int_{-\infty}^{+\infty}\frac{\mu_0 ndxir^2}{2\left(r^2+x^2\right)^{3/2}}.$$

This integral is readily evaluated by making the substitution

$$x=r\tan\theta,$$
$$dx=r\sec^2\theta d\theta.$$

We find

$$B(P)=\frac{\mu_0 ni}{2}\int_{-\pi/2}^{+\pi/2}\cos\theta d\theta=\frac{\mu_0 ni}{2}\times 2=\mu_0 ni.$$

The magnetic field at the axis of the solenoid is, therefore,

$$B = \mu_0 n i.$$