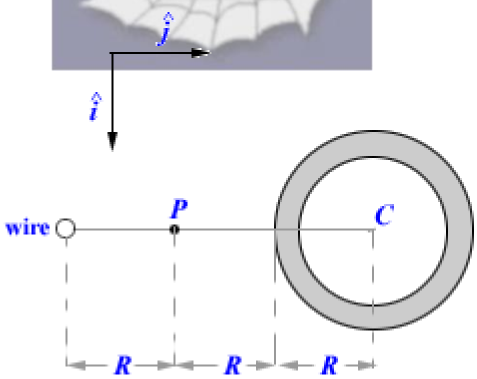
478.

<u>Problem 35.49 (RHK)</u>

A *long cylindrical pipe, with an outside radius of R, carries a* (*uniformly distributed*) *current of* i_0 (*into the paper as shown in the figure*). *A wire runs parallel to the pipe at a distance* $3R$ *from the centre to the centre. We have to calculate the magnitude and direction of the current in the wire that would cause the resultant magnetic field at the point P to have the same magnitude, but the opposite direction, as the resultant field at the centre of the pipe.*



<u>Solution:</u>

It is given that a long circular pipe, with an outside radius R, carries a uniformly distributed current i_0 into the paper. A wire runs parallel to the pipe at a distance $3R$ from the centre of the pipe. We have to calculate the magnitude and direction of the current in the wire that would cause the resultant magnetic field at the point P to have the same magnitude, but the opposite direction, as the resultant field at the centre of the pipe.

As the current in the conductor is into the paper, the magnetic field at P due to the current i_0 in the circular pipe will be

$$\vec{B}_{cond}\left(P\right)=\frac{\mu_0 i_0}{4\pi R}\left(-\hat{i}\right).$$

The magnetic field due to the current in the pipe at its centre will be zero; this follows from the cylindrical symmetry of the current flow in the pipe.

Let the current in the wire be i_{wire}. The magnitude of the magnetic field at the centre of the pipe due to the current in the wire will be

$$B_{wire}\left(C\right)=\frac{\mu_0 i_{wire}}{2\pi\times 3R}=\frac{\mu_0 i_{wire}}{6\pi R}.$$

As the resultant magnetic field at P has the same magnitude but opposite direction to the field at C, the current flow in the wire should be into the paper so that the direction of the magnetic field at the centre of the long circular pipe can be made to be opposite to that of the resultant field at P. That is

$$\vec{B}_{wire}(C) = \frac{\mu_0 i_{wire}}{6\pi R}\hat{i}.$$

The magnetic field at P due to the wire will be

$$\vec{B}_{wire}(P) = \frac{\mu_0 i_{wire}}{2\pi R}\hat{i}.$$

The resultant magnetic field at P will be

$$\vec{B}(P) = \vec{B}_{wire}(P) + \vec{B}_{cond}(P)$$
$$= \left(-\frac{\mu_0 i_{wire}}{2\pi R} + \frac{\mu_0 i_0}{4\pi R}\right)\left(-\hat{i}\right).$$

It is given that the magnitude of the magnetic field at C is the same as the magnitude of the magnetic field at P.

This condition implies that

$$\frac{\mu_0}{4\pi R}\left(i_0 - 2i_{wire}\right) = \frac{\mu_0 i_{wire}}{6\pi R},$$

or

$$i_{wire} = \frac{3}{8} i_0 \ .$$

The current $i_{wire} = \frac{3}{8} i_0$ flows in the wire in the direction of into the page.