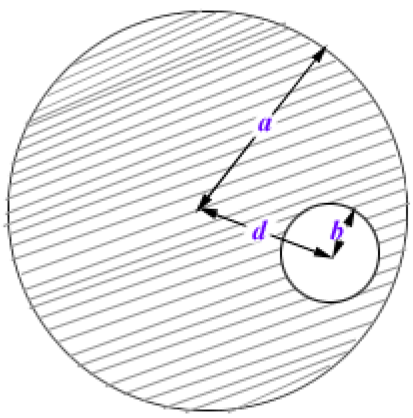
**477.**

**<u>Problem 35.48 (RHK)</u>**

*In the figure a cross-section has been shown of a long cylindrical conductor of radius* $a$ *containing a long cylindrical hole of radius* $b$. *The axes of the two cylinders are parallel and are a distance* $d$ *apart. A current* $i$ *is uniformly distributed over the cross-hatched area in the figure.* (a) *Using the superposition ideas we have to show that the magnetic field at the centre of the hole is*

$$B = \frac{\mu_0 i d}{2\pi\left(a^2 - b^2\right)}.$$

(b) *We have to discuss the two special cases* $b = 0$ *and* $d = 0$.



**Solution:**

A long cylindrical conductor of radius $a$ has a long cylindrical hole of radius *b*. The axes of the two cylinders are parallel and are at a distance *d* apart. A current $i$ is uniformly distributed over the cross-hatched area, as shown in the figure. The current density is

$$j = \frac{i}{\pi\left(a^2 - b^2\right)}.$$

(a)

We will find the field at the centre of the cylinder of radius *b* by using the superposition principle. We assume that the situation of current flow in the conductor is equivalent to current flowing in the full cross-section of the cylinder of radius *a*, that is without the cylindrical hole of radius *b*, and the flow of current of the same density in the opposite direction in the cylinder of radius *b*. The magnetic field at the axis of the cylindrical hole will be the vector sum of the magnetic field due to each of the two flows of the currents.

We know that the magnetic field at the centre of a long cylindrical current with uniform current density is zero. Therefore, the magnetic field at the axis of the cylindrical hole will be the field at a distance *d* from the axis of the cylindrical conductor of radius *d* with current density

$$j=\frac{i}{\pi\left(a^2-b^2\right)}.$$

The magnetic field at *d* will, therefore, be

$$B(d)=\frac{\mu_0}{2\pi d}\frac{i\times\pi d^2}{\pi\left(a^2-b^2\right)}=\frac{\mu_0 id}{2\pi\left(a^2-b^2\right)}.$$

(b)

For $b=0$,

$$B(d)=\frac{\mu_0}{2\pi d}\frac{i\times\pi d^2}{\pi}a^{-2}=\frac{\mu_0 id}{2\pi a^2}.$$

For $d=0$,

$B=0$.

(c)

We will next show that the magnetic field in the hole is uniform. Let us consider a point inside the hole on the line joining the centres of the cylinders of radii $a$ and $b$, which is at a distance $\xi$ from the centre of the cylinder of radius $b$.

By superposition principle the magnetic field at this point will be the sum of the field due to the current enclosed in the cylinder of radius $\left(d-\xi\right)$ and the field due to the current enclosed in the cylinder of radius $\xi$ but flowing in the opposite direction.

Therefore,

$$B(d-\xi) = \frac{\mu_0 j}{2\pi}\left(\frac{\pi(d-\xi)^2}{(d-\xi)} + \frac{\pi\xi^2}{\xi}\right) = \frac{\mu_0 j d}{2}$$
$$= \frac{\mu_0 i d}{2\pi\left(a^2 - b^2\right)}.$$

We note that it is independent of the point inside the hole on the line joining the centres of radii *a* and *b*.