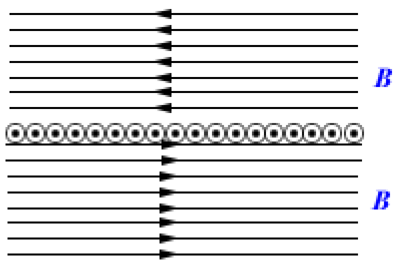
476.

Problem 35.46 (RHK)

A conductor consists of an infinite number of adjacent wires, each infinitely long and carrying a current i_0. We have to show that lines of $\vec{B}$ are as represented in the figure and that B for all points above and below the infinite current sheet is given by

$$B = \frac{1}{2}\mu_0 n i_0,$$

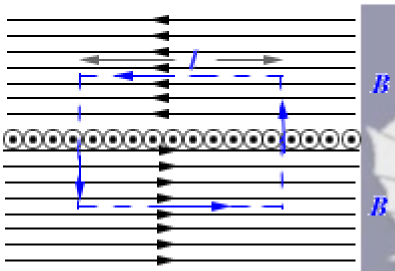
where n is the number of wires per unit length. We will derive this result both by direct application of Ampere's law and by using the Biot-Savart law.



Solution:

We will first solve the problem by applying Ampere's law. The direction of the magnetic field can be determined by using the fact that the field due to infinitely long wires in which current is coming out of the page will be in the form of concentric circles drawn counter-clockwise. As the conductor consists of an infinite number of adjacent long wires, the magnetic field will be parallel and their direction above and below the current sheet will be as shown in the figure.

For calculating the magnetic field at a distance *r* from the sheet we consider an Amperian loop as shown in the figure below.



Because of the symmetry the magnitude of the magnetic field above and below the sheet at the same distance will be equal, and the components of the field along the lines perpendicular to the sheet will be zero.

We apply the Ampere's law,

$$\oint \vec{B}.d\vec{s} = \mu_0(\text{total current enclosed}),$$

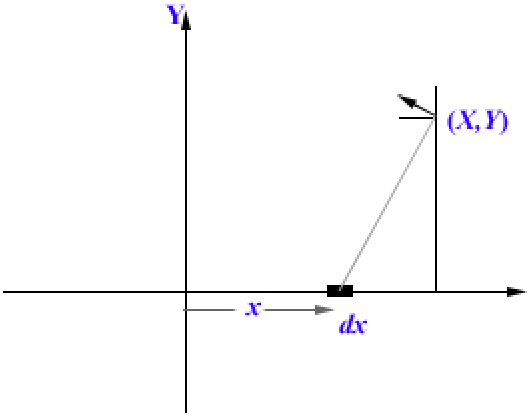
using the Amperian loop shown in the figure. We have

$$2lB(r) = \mu_0 nli,$$

and

$$B(r) = \frac{\mu_0 ni}{2}.$$

The field is independent of r and is parallel, as shown in the figure.



We will calculate the magnetic field at (X, Y) due to the infinite current sheet. Consider a current element $ni_0 dx$, which is of infinite length in the *z*-direction, as shown in the figure. Component of the magnetic field at (X, Y) parallel to the *x*-axis will be

$$dB_{\text{P}}(X,Y) = \frac{\mu_0 n i_0 dx}{2\pi\left((X-x)^2+Y^2\right)^{1/2}} \frac{Y}{\left((X-x)^2+Y^2\right)^{1/2}}$$

$$= \frac{\mu_0 n i_0 Y dx}{2\pi\left((X-x)^2+Y^2\right)}.$$

Therefore, the magnetic field at (X, Y) due to the entire infinite current sheet will be obtained by integrating dB_{P} with respect to *x* from $-\infty$ to $+\infty$. We have

$$B_{\text{P}}(X,Y) = \int_{-\infty}^{\infty} \frac{\mu_0 n i_0 Y dx}{2\pi\left((X-x)^2+Y^2\right)}.$$

We make the substitution

$$X - x = Y \tan\theta,$$

$$-dx = Y \sec^2 \theta d\theta.$$

We have

$$B_{\mathrm{P}}(X,Y) = \int_{-\pi/2}^{\pi/2} \frac{\mu_0 n i_0 d\theta}{2\pi} = \frac{\mu_0 n i_0}{2}.$$

Note that by symmetry, the perpendicular components of $B\left(X,Y\right)$ will cancel in pairs. We, therefore, have

$$B\left(X,Y\right) = B_{\mathrm{P}}\left(X,Y\right) = \frac{\mu_0 n i_0}{2}.$$