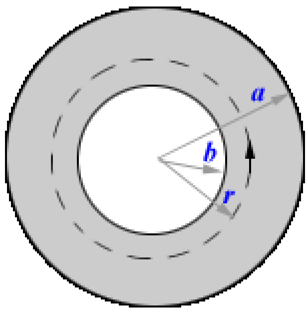
474.

Problem 35.44 (RHK)

In the figure a cross-section has been shown of a hollow cylindrical conductor of radii a *and* b. *It carries a uniformly distributed current* i. (a) *Using the circular Amperian loop shown in the figure, we have to show that* $B(r)$ *for the range* $b < r < a$ *is given by*

$$B(r) = \frac{\mu_0 i}{2\pi\left(a^2 - b^2\right)} \frac{r^2 - b^2}{r}.$$

(b) *We have to test this formula for the special cases of* $r = a$, $r = b$, *and* $b = 0$.



Solution:

We are considering a hollow cylindrical conductor of *radii* a *and* b, which carries a uniformly distributed current i. Therefore, the current density

$$j = \frac{i}{\pi\left(a^2 - b^2\right)}.$$

Because of the cylindrical symmetry the magnetic field inside the conductor will be circular and will depend on the distance from the axis of the cylinder.

We consider an Amperian loop of radius *r* as shown in the diagram.

We now apply the Ampere's law and write

$$\oint \vec{B}(r).d\vec{s} = \mu_0 \times \text{current enclosed.}$$

$$\vec{B}(r).d\vec{s} = B(r)rd\theta.$$

$$\therefore \quad \oint \vec{B}(r).d\vec{s} = 2\pi r B(r) = \mu_0 j \pi\left(r^2 - b^2\right),$$

or

$$B(r) = \frac{\mu_0 i}{2\pi\left(a^2 - b^2\right)} \frac{\left(r^2 - b^2\right)}{r}.$$

We will now test this formula for different special cases.

(a)

For $r = a$,

$$B(a) = \frac{\mu_0 i}{2\pi a}.$$

It is the magnetic field outside a wire of radius *a* carrying current i.

(b)

For $r=b,$

$B(b)=0.$

There is zero magnetic field at the surface $r=b,$ and within the hollow part of the cylindrical conductor.

(c)

For $b=0,$

$$B(r)=\frac{\mu_0 ir}{2\pi a^2}.$$