471.

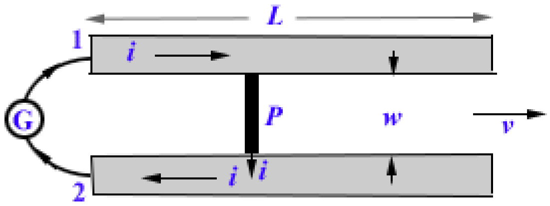
Problem 35.37 (RHK)

In the figure an idealised schematic of an "electromagnetic rail gun" has been shown. The projectile P sits between and in contact with two parallel rails along which it can slide. A generator G provides a current that flows up one rail, across the projectile, and back down the other rail.

Let w *the distance between the rails,* r *the radius of the rails* (*presumed circular*), *and* i *the current. We have to show that the force on the projectile is to the right and given approximately by*

$$F = \frac{1}{2}\left(\frac{i^2\mu_0}{\pi}\right)\ln\left(\frac{w+r}{r}\right).$$

(b) *If the projectile* (*in this case a test slug*), *starts from the left end of the rail at rest, we have to find the speed v at which it is expelled at right. We may assume that* $i = 450\ \text{kA}$, $w = 12\ \text{mm}$, $r = 6.7\ \text{cm}$, $L = 4.0\ \text{m}$, *and that the mass of the slug is* $m = 10.0\ \text{g}$.

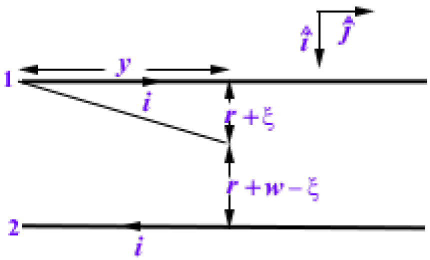


Solution:

(a)

In the figure an idealised schematic of an "electromagnetic rail gun" has been shown. The projectile *P* sits between and in contact with two parallel rails along which it can slide. We assume that the rails are circular and of radius *r*. Let the current that flows through rail 1 and then through the projectile to the rail 2 be i. We will first calculate the magnetic field in the space between the two rails when current i is flowing through them. We will approximate the rails as current carrying semi-infinite lines.

We will use Biot-Savart law for calculating the magnetic field. We draw a line diagram for calculating the magnetic field at a distance $r+\xi$ from the axis of the rail 1.



$$\vec{B}(r+\xi)=\left(\frac{\mu_0}{4\pi}\right)\left(-\hat{k}\right)\int_0^{\infty}\frac{idy}{\left(y^2+(r+\xi)^2\right)}\frac{(r+\xi)}{\left(y^2+(r+\xi)^2\right)^{1/2}}$$

$$+\left(\frac{\mu_0}{4\pi}\right)\left(-\hat{k}\right)\int_0^{\infty}\frac{idy}{\left(y^2+(r+w-\xi)^2\right)}\frac{(r+w-\xi)}{\left(y^2+(r+w-\xi)^2\right)^{1/2}}.$$

Integrals are of standard form and many integrals of this type have been calculated in other problems, so we write the answer,

$$\vec{B}(r+\xi)=\left(\frac{\mu_0 i}{4\pi}\right)\left(-\hat{k}\right)\left(\frac{1}{r+\xi}+\frac{1}{r+w-\xi}\right).$$

We calculate next the force on the projectile. The current flowing through the projectile is i and its direction is from the rail 1 toward the rail 2, that is along $\hat{i}$. We consider an infinitesimal current element $id\xi\hat{i}$ at a distance ξ from the rail1. Lorentz force on this element is

$$d\vec{F}=id\xi\hat{i}\times\vec{B}(r+\xi)$$

$$=id\xi\hat{i}\times\left(\frac{\mu_0 i}{4\pi}\right)\left(-\hat{k}\right)\left(\frac{1}{r+\xi}+\frac{1}{r+w-\xi}\right)$$

$$=\left(\frac{\mu_0 i^2}{4\pi}\right)\hat{j}\left(\frac{1}{r+\xi}+\frac{1}{r+w-\xi}\right)d\xi.$$

Therefore, the force on the projectile can now be obtained by integrating $d\vec{F}$. We have

$$\vec{F} = \int_0^w \left(\frac{\mu_0 i^2}{4\pi}\right)\hat{j}\left(\frac{1}{r+\xi}+\frac{1}{r+w-\xi}\right)d\xi.$$

And

$$\vec{F} = \left(\frac{\mu_0 i^2}{2\pi}\right)\hat{j}\ln\left(\frac{r+w}{w}\right).$$

(b)

We will use the following data in answering the second part of the problem.

$i = 450\text{ kA}$,

$w = 12\text{ mm}$,

$r = 6.7\text{ cm}$,

$L = 4.0\text{ m}$,

and the mass of the projectile, $m = 10.0\text{ g}$.

From the data, we calculate the value of the force on the projectile,

$$F = \left(10^{-7}\right)\times 2\times\left(450\times 10^3\right)^2\times\ln\left(\frac{6.7+1.2}{6.7}\right)\text{ N}$$

$$= 6.67\times 10^3\text{ N}.$$

Acceleration of the projectile due to the "electromagnetic rail gun" will be

$$a = \frac{F}{m} = \frac{6.67 \times 10^{3}}{10 \times 10^{-3}} \text{ m s}^{-2}.$$

As the projectile starts from the left-hand end of the rails with zero velocity, the velocity with which it will be expelled from the "electromagnetic rail gun" after travelling its length, which is 4.0 m, will be given by the relation

$$v^{2} = 2aL = 2 \times 6.67 \times 10^{5} \times 4 \ \left(\text{m s}^{-1}\right)^{2},$$

and

$$\therefore \ v = 2.3 \text{ km s}^{-1}.$$