**469.**

## Problem 35.32 (RHK)

*A thin plastic disk of radius R has a charge q uniformly distributed over its surface. If the disk rotates at an angular frequency ω about its axis, we have to show that* (a) *the magnetic field at the centre of the disk is*

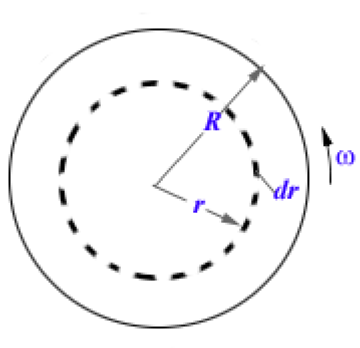
$$B = \frac{\mu_0 \omega q}{2\pi R},$$

*and the magnetic dipole moment of the disk is*

$$\mu = \frac{\omega q R^2}{4}.$$

## Solution:

(a)



A thin plastic disk of radius R has a charge q uniformly distributed over its surface. The charge per unit area on the disk will be

$$\sigma = \frac{q}{\pi R^2}.$$

The disk is rotating about its axis with angular velocity $\omega$. Charge on the disk will also rotate with angular velocity $\omega$. The current on the disk can be calculated by assuming that the rotating disk is equivalent to an array of current loops. For calculating the current arising out of the rotation of the disk and the magnetic field produced, we consider a ring of radius *r* and width $dr$. As the disk is rotating with angular frequency $\omega$, the rotating charge on this ring, $\left(2\pi r dr q / \pi R^2\right)$, effectively behaves like a current in a ring of radius r of magnitude

$$i(r) = \frac{2qrdr}{R^2}\frac{\omega}{2\pi} = \frac{q\omega r dr}{\pi R^2}.$$

The magnetic field at the centre due to this ring will be

$$\frac{\mu_0}{2r}(current) = \frac{\mu_0}{2r}\frac{q\omega r dr}{\pi R^2} = \frac{\mu_0 q\omega dr}{2\pi R^2}.$$

The magnetic field at the centre of the rotating disk can be calculated by integrating the above expression from 0 to *R*. We have for the magnetic field at the centre

$$B = \int_0^R \frac{\mu_0 q\omega dr}{2\pi R^2} = \frac{\mu_0 q\omega}{2\pi R}.$$

(b)

We calculate next the magnetic dipole moment of the rotating uniformly charged disk. The magnetic dipole moment of the disk can be calculated by adding contribution to the dipole moment from each circular ring of rotating charges. The magnetic dipole moment of due to the ring of radius *r*, width $dr$, is current multiplied by the area enclosed. That is

$$\Delta\mu(r) = \pi r^2 \frac{q\omega r dr}{\pi R^2} = \frac{q\omega r^3 dr}{R^2}.$$

Therefore, the magnetic dipole moment due to the rotating uniformly charged disk will be

$$\mu = \int_0^R \frac{q\omega r^3 dr}{R^2} = \frac{q\omega R^2}{4}.$$