468.

<u>Problem 35.29 (RHK)</u>

(a) *A wire in the form of a regular polygon of* n *sides is just enclosed by a circle of radius* a*. If the current in the wire is* i*, we have to show that the magnetic field* $\vec{B}$ *at the centre of the circle is given in magnitude by*

$$B = \frac{\mu_0 n i}{2\pi a}\tan\left(\pi/n\right).$$

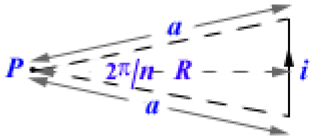
(b) *We have to show that as* $n \to \infty$ *this result approaches that of a circular loop*. (c) We have to find the dipole moment of the polygon.

<u>Solution:</u>

(a)

A wire in the form of a regular polygon of n sides is just enclosed by a circle of radius a. The angle subtended by each side of the polygon with the centre of the circle of radius a will be $2\pi/n$. Length of each side of the polygon will be

$$L = 2a\sin\left(\frac{\pi}{n}\right).$$



We have already shown that the magnetic field B associated with a current carrying straight line segment of length L at a distance R from the segment along a perpendicular bisector is

$$B = \frac{\mu_0 i}{2\pi R}\frac{L}{\left(L^2 + 4R^2\right)^{1/2}}.$$

For our problem we use that contribution to the magnetic field at the point P by each side of the polygon will be the same and can be obtained from the above result for the values

$$L = 2a\sin\left(\frac{\pi}{n}\right),$$

$$R = a\cos\left(\frac{\pi}{n}\right).$$

Therefore, the magnitude of the magnetic field at P by each side of the polygon will be

$$\frac{\mu_0 i}{2\pi a \cos\left(\frac{\pi}{n}\right)} \times \frac{2a \sin\left(\frac{\pi}{n}\right)}{\left(4a^2 \sin^2\left(\frac{\pi}{n}\right) + 4a^2 \cos^2\left(\frac{\pi}{n}\right)\right)^{1/2}}$$

$$= \frac{\mu_0 i}{\pi} \frac{\tan\left(\frac{\pi}{n}\right)}{2a}.$$

As the magnetic field at the centre of the polygon sue to each side will be perpendicular to the plane of the polygon, the total magnetic field will be n times the field due to one side. Therefore,

$$B = \frac{\mu_0 n i}{2\pi a} \tan(\pi/n).$$

(b)

As

$$n \to \infty,\ n \tan\left(\frac{\pi}{n}\right) = \lim_{n\to\infty} \frac{n \sin\left(\frac{\pi}{n}\right)}{\cos\left(\frac{\pi}{n}\right)} = \pi.$$

$$\therefore\ \lim_{n\to\infty} B(n) = \frac{\mu_0 i}{2a}.$$

It is same as the magnetic field at the centre of a circular loop of radius a carrying current i.

(c)

We calculate next the dipole moment of the polygon. The magnetic dipole moment of a planar closed loop in current times the area enclosed. The area of each triangle formed by joining the side of a polygon with the centre will be

$$\frac{A}{n} = \left(a \sin\left(\frac{\pi}{n}\right)\right)\left(a \cos\left(\frac{\pi}{n}\right)\right) = \frac{a^2 \sin\left(\frac{2\pi}{n}\right)}{2},$$

$$\therefore\ A = \frac{na^2 \sin\left(\frac{2\pi}{n}\right)}{2}.$$

And, the magnetic dipole moment of the polygon will be

$$\mu = iA = \frac{ia^2 n}{2} \sin\left(\frac{2\pi}{n}\right).$$