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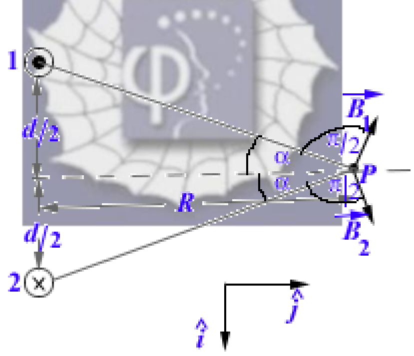
<u>Problem 35.24 (RHK)</u>

Two long wires a distance d apart equal antiparallel currents i, *as shown in the figure.* (a) *We have to show that the magnetic field strength at P, which is equidistant from the wires, is given by*

$$B = \frac{2\mu_0 i d}{\pi\left(4R^2 + d^2\right)}.$$

(b) *We have to determine the direction of* $\vec{B}$.



<u>Solution:</u>

In the figure two long current carrying wires have been shown. Each wire carries a current i. In wire 1 the current is coming out of the page and in wire 2 the current is going into the page. Separation between the two wires is d. We will calculate the magnetic field at point P, which is at a distance R from the mid-point of the line joining the two wires. We have fixed the coordinate system by indicating the unit vectors $\hat{i}$ and $\hat{j}$; $\hat{i}\times\hat{j}=\hat{k}$.

The magnetic field at P due to the wire 1 will be $\perp$ to the line joining 1 to P, as shown in the figure. Using the expression for the magnetic field due to a long current carrying wire, we have

$$\vec{B}_1=\frac{\mu_0 i}{2\pi\left(R^2+d^2/4\right)^{1/2}}\left(-\cos\alpha\hat{i}+\sin\alpha\hat{j}\right),$$

$$\sin\alpha=\frac{d/2}{\left(R^2+d^2/4\right)^{1/2}}.$$

The magnetic field at P due to the wire 1 will be $\perp$ to the line joining 1 to P, as shown in the figure. Using the expression for the magnetic field due to a long current carrying wire, we have

$$\vec{B}_2=\frac{\mu_0 i}{2\pi\left(R^2+d^2/4\right)^{1/2}}\left(+\cos\alpha\hat{i}+\sin\alpha\hat{j}\right).$$

Therefore, the magnetic field at P due to the current carrying wires will be

$$\vec{B} = \vec{B}_1 + \vec{B}_2 = \frac{\mu_0 i}{2\pi\left(R^2 + d^2/4\right)^{1/2}}\left(-\cos\alpha\hat{i} + \sin\alpha\,\hat{j}\right)$$

$$+\frac{\mu_0 i}{2\pi\left(R^2 + d^2/4\right)^{1/2}}\left(+\cos\alpha\hat{i} + \sin\alpha\,\hat{j}\right)$$

$$= \frac{\mu_0 i}{2\pi\left(R^2 + d^2/4\right)^{1/2}} 2\sin\alpha\,\hat{j}.$$

Substituting the expression for $\sin\alpha$, we find

$$\vec{B} = \frac{2\mu_0 i d}{\pi\left(4R^2 + d^2\right)}\hat{j}.$$

(b)

The direction of the magnetic field at P is along the vector $\hat{j}$, which is in the direction of the line $\perp$ to the line joining the two wires at the middle, as shown in the figure.