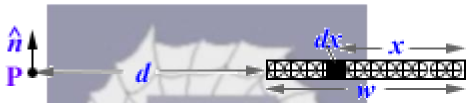
464.

<u>Problem 35.21 (RHK)</u>

In the figure cross-section has been shown of a long thin ribbon of width w *that is carrying a uniformly distributed total current* i *into the page. We have to calculate the magnitude and the direction of the magnetic field* $\vec{B}$ *at a point* P *in the plane of the ribbon at a distance d from its edge.*



<u>Solution:</u>

We imagine the ribbon to be comprising of many, long, thin, parallel wires each of thickness dx. The current in each such wire will be idx/w. As the current is entering into the plane of page, using the right-hand rule, we note that the direction of the magnetic field at P will be in the direction of the unit vector $\hat{n}$, and the magnetic field will be uniform circles around each thin wire.

The magnetic field at P due to a wire element dx at distance x from the right-hand edge will be

$$\Delta \vec{B}(P) = \frac{\mu_0 i dx}{w\left(2\pi\left(d + w - x\right)\right)}\hat{n}.$$

Therefore, the magnetic field at P due to the ribbon with current i will be given by the integral

$$\int_0^w \frac{\mu_0 i dx}{w\left(2\pi\left(d + w - x\right)\right)}\hat{n} = \frac{\mu_0 i}{2\pi w}\hat{n}\int_0^w \frac{dx}{\left(d + w - x\right)}.$$

For calculating the above integral, we make the substitution

$$d + w - x = \xi,$$
$$-dx = d\xi.$$

We get

$$\vec{B}(P) = \frac{\mu_0 i}{2\pi w}\hat{n}\int_d^{d+w} \frac{d\xi}{\xi} = \frac{\mu_0 i}{2\pi w}\hat{n}\ln\left(\frac{d + w}{d}\right).$$