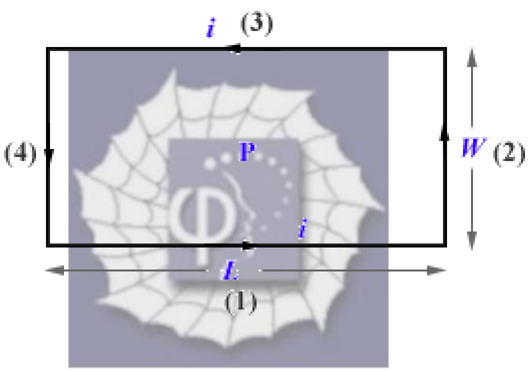
462.

<u>Problem 35.17 (RHK)</u>

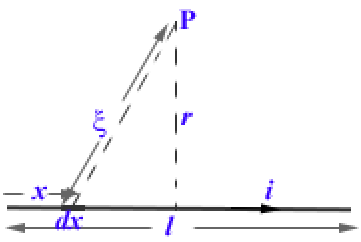
We have to show that B at the centre of a rectangular loop of wire of length L and width W, carrying a current i, *is given by*

$$B = \frac{2\mu_0 i}{\pi}\frac{\left(L^2 + W^2\right)^{1/2}}{LW}.$$



<u>Solution:</u>

We first calculate the magnetic field due to a straight wire segment of length *l*, carrying current i, at a distance *r* from its mid-point.



Using Biot-Savart law we write the expression for the magnetic field at P due to an infinitesimal current element $i d\vec{x}$.

$$d\vec{B} = \frac{\mu_0}{4\pi} \frac{idx\xi}{\xi^3} \frac{r}{\xi} \hat{k},$$

$$\xi = \left(r^2 + \left(l/2 - x\right)^2\right)^{1/2} \; .$$

$$\therefore \; \vec{B} = \frac{\mu_0 ir}{4\pi} \hat{k} \int_0^l \frac{dx}{\left(r^2 + \left(l/2 - x\right)^2\right)^{3/2}}.$$

In the integral, we make the substitution

$$\frac{l}{2} - x = r \tan\theta,$$

$$-dx = r \sec^2 \theta d\theta.$$

We get

$$\vec{B} = \frac{\mu_0}{4\pi} ir\hat{k} \int_{-\tan^{-1}(l/2r)}^{\tan^{-1}(l/2r)} \frac{\cos\theta d\theta}{r^2} = \frac{\mu_0}{2\pi} \frac{i}{r} \hat{k} \frac{l}{\left(l^2 + 4r^2\right)}.$$

We will use this result and write contributions to $\vec{B}$ at P due to the current carrying line segments (1), (2), (3) and (4).

$$\vec{B}_{(1)} = \vec{B}_{(3)} = \frac{\mu_0}{2\pi} \frac{i\hat{k}}{W/2} \frac{L}{\left(L^2 + W^2\right)^{1/2}},$$

$$\vec{B}_{(2)} = \vec{B}_{(4)} = \frac{\mu_0}{2\pi} \frac{i\hat{k}}{L/2} \frac{L}{\left(L^2 + W^2\right)^{1/2}}.$$

$$\therefore\ \vec{B} = \frac{2\mu_0 i}{\pi}\frac{\hat{k}}{\left(L^2+W^2\right)^{1/2}}\left(\frac{L}{W}+\frac{W}{L}\right) = \frac{2\mu_0 i}{\pi}\frac{\left(L^2+W^2\right)^{1/2}}{LW}\hat{k}.$$