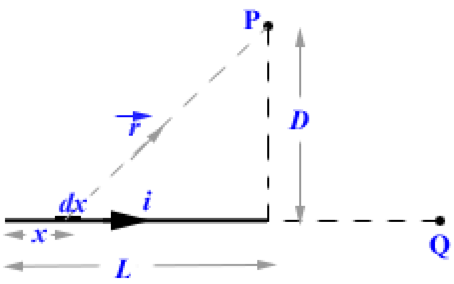
**460.**

**Problem 35.14 (RHK)**

*A straight section of wire of length L carries a current* $i$. (a) *We have to show that the magnetic field associated with this segment at* P, *a perpendicular distance D from one end of the wire* (*see figure*), *is given by*

$$B = \frac{\mu_0 i}{4\pi D}\frac{L}{\left(L^2 + D^2\right)^{1/2}}.$$

(b) *We have to show that the magnetic field is zero at point Q, along the line of the wire.*




**Solution:**

Using the Biot-Savart law we write the expression for the magnetic field at the point P due to an infinitesimal segment of wire of length $d\vec{x}$ carrying current $i$. We have

$$d\vec{B} = \frac{\mu_0}{4\pi}\frac{i d\vec{x} \times \vec{r}}{r^3} = \frac{\mu_0}{4\pi}\frac{idx}{r^2}\frac{D}{r}\hat{k},$$

where $\hat{k}$ is a unit vector perpendicular to the plane of the figure.

But

$$r = \left( (L-x)^2 + D^2 \right)^{1/2}.$$

Therefore,

$$\vec{B}(\mathrm{P}) = \frac{\mu_0 i D}{4\pi}\hat{k}\int_0^L \frac{dx}{\left( (L-x)^2 + D^2 \right)^{3/2}}.$$

We make the following substitution in the integral

$$L - x = D\tan\theta,$$

$$-dx = D\sec^2\theta.$$

We get

$$\vec{B}(\mathrm{P}) = \frac{\mu_0 i D}{4\pi}\hat{k}\int_0^L \frac{dx}{\left( (L-x)^2 + D^2 \right)^{3/2}}$$

$$= \frac{\mu_0 i D}{4\pi}\hat{k}\int_0^{\tan^{-1}\left(\frac{L}{D}\right)} \frac{D\sec^2\theta d\theta}{D^3\sec^3\theta} = \frac{\mu_0 i D}{4\pi}\hat{k}\int_0^{\tan^{-1}\left(\frac{L}{D}\right)} \frac{\cos\theta d\theta}{D}$$

$$= \frac{\mu_0 i}{4\pi}\hat{k}\left[\sin\theta\right]_0^{\tan^{-1}\left(\frac{L}{D}\right)} = \frac{\mu_0 i}{4\pi}\hat{k} \times \frac{L}{\left(L^2 + D^2\right)^{\frac{1}{2}}},$$

where we have used

$$\sin\left(\tan^{-1}\left(\frac{L}{D}\right)\right) = \frac{L}{\left(L^2 + D^2\right)^{\frac{1}{2}}}.$$

(b)

As the point Q is along the line segment carrying the current $i$, from the Biot-Savart law, we note that as the vectors $\vec{i}$ and $\vec{r}$ are parallel, their cross product is zero. Hence the magnetic field due to the line segment will be zero at Q.