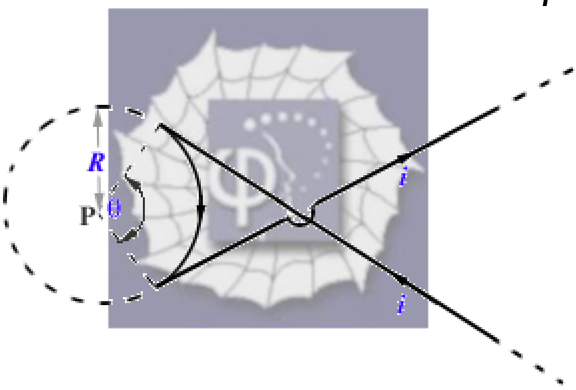
459.

Problem 35.13 (RHK)

A wire carrying current i has the configuration as shown in the figure. Two semi-infinite straight sections, each tangent to the same circle, are connected by a circular arc, of angle θ, along the circumference of the circle, with all sections lying in the same plane. We have to find θ for B to be zero at the centre of the circle.

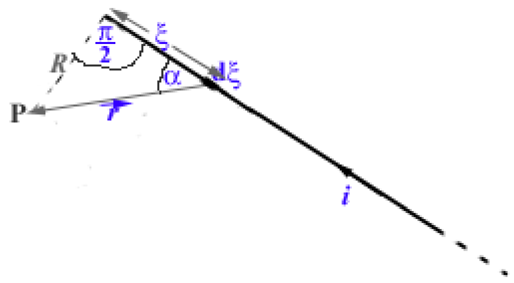


Solution:

Let the wire sections carrying current i be in the xy plane and the z-axis be $\perp$ to the xy-plane with orientation $\hat{i} \times \hat{j} = \hat{k}$.

Using the Biot-Savart law we will calculate the magnetic field at the point P by the three sections of the current carrying wire.

We will first calculate the magnetic field at P due to the semi-infinite length section shown in the following figure.



Let us write the field due to the element as $d\vec{\xi}$ at a distance ξ from the open end of the semi-infinite line.

$$d\vec{B}_1 = \frac{\mu_0}{4\pi}(-i)\frac{d\vec{\xi}\times\vec{r}}{r^3} = \frac{\mu_0 i}{4\pi}\frac{d\xi}{r^2}\sin\alpha\hat{k} = \frac{\mu_0 i}{4\pi}\frac{R d\xi}{\left(R^2+\xi^2\right)^{3/2}}\hat{k}.$$

And

$$\vec{B}_1 = \frac{\mu_0 i R}{4\pi}\hat{k}\int_0^\infty \frac{d\xi}{\left(R^2+\xi^2\right)^{3/2}}.$$

We calculate next the integral

$$\int_0^\infty \frac{d\xi}{\left(R^2+\xi^2\right)^{3/2}}.$$

Making the substitution

$$\xi = R\tan\theta,$$

$$d\xi = R\sec^2\theta d\theta,$$

we have

$$\int_0^\infty \frac{d\xi}{\left(R^2+\xi^2\right)^{3/2}} = \int_0^{\pi/2} \frac{R\sec^2\theta d\theta}{R^3\sec^3\theta} = \frac{1}{R^2}\int_0^{\pi/2}\cos\theta d\theta = \frac{1}{R^2}.$$

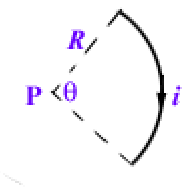
Therefore,

$$\vec{B}_1 = \frac{\mu_0 i}{4\pi R}\hat{k}.$$

Similarly, the magnetic field at P due to the other semi-infinite wire carrying current i as shown in the first figure will also be

$$\vec{B}_2 = \frac{\mu_0 i}{4\pi R}\hat{k}.$$

We next calculate the magnetic field at P due to the circular arc subtending an angle θ with P.



From the Biot-Savart law

$$d\vec{B} = \frac{\mu_0}{4\pi}\frac{i d\vec{s}\times\vec{r}}{r^3}.$$

As the radial direction is perpendicular to the tangent at the circumference,

$$d\vec{s}\times\vec{r} = rds\left(-\hat{k}\right).$$

The magnetic field at P due to the arc of angle θ will be

$$\vec{B}_{arc} = -\frac{\mu_0 i}{4\pi}\times\frac{R\theta}{R^2}\hat{k}.$$

The magnetic field at P due all the three sections of the wire carrying current i will, therefore, be

$$\vec{B} = \frac{2\mu_0 i}{4\pi R}\hat{k} - \frac{\mu_0 i\theta}{4\pi R}\hat{k}.$$

For $\vec{B}$ to be zero

$\theta = 2$ rad.