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## Problem 35.9 (RHK)

A length of wire carrying a current $i$ is bent into $a$ circular coil of one turn, as shown in fig a. The same length of wire has been bent more sharply, to give a double loop of smaller radius, as shown in fig b. (a) If $B_{a}$ and $B_{b}$ are magnitudes of the magnetic fields at the centres of the two loops, we have to find the ratio $B_{b} / B_{a}$. (b) We have to find the ratio of their dipole moments, $\mu_{b} / \mu_{a}$.


## Solution:

(a)

As shown in the figure a, the length of the wire is bent into a circular loop of one coil, and its radius is $R$.
The magnetic field at the centre of the current carrying circular loop will be perpendicular to the plane of the
loop and its magnitude can be calculated using the BiotSavart law and the symmetry of the problem. As the magnetic field produced by a current element $i d{ }^{t}$ at a point ${ }_{r}$ from the current element is given by
$d \stackrel{\mathrm{r}}{B}=\frac{\mu_{0}}{4 \pi} \frac{i d^{1}{ }^{1} \times \frac{1}{r}}{r^{3}}$,
we have
$B_{a}=\frac{\mu_{0}}{4 \pi} \times \frac{2 \pi R i}{R^{2}}=\frac{\mu_{0} i}{2 R}$.
As shown in fib b , when the length of wire is bent into a circular coil of two loops its radius $r$ will be related to $R$ as
$4 \pi r=2 \pi R$,
or
$r=R / 2$.
As the second coil has two turns of radius $r$, the magnetic field at its centre when current $i$ is flowing through it, as shown in fig b , will be

$$
B_{b}=2 \times\left(\frac{\mu_{0} i}{2 r}\right)=\frac{2 \mu_{0} i}{R} .
$$

Therefore,
$B_{b} / B_{a}=4$.
(b)

The magnetic dipole moment of a planar-loop carrying current $i$ is
$\mu=i A n$,
where $n$ is the number of times the current carrying wire encloses the area $A$.

We, therefore, have
$\mu_{a}=\pi R^{2} i$,
and
$\mu_{b}=2\left(\pi r^{2}\right) i=\frac{\pi R^{2} i}{2}$.
Therefore,
$\frac{\mu_{b}}{\mu_{a}}=\frac{1}{2}$.

