455.

Problem 34.33 (RHK)

We have to estimate the total path length travelled by a deuteron during the acceleration process. We may assume an accelerating potential between the dees of 80 kV, *a dee radius of* 53 cm, *and an oscillator frequency of* 12 MHz.

Solution:

A deuteron nucleus has the same charge as that of a proton and nearly twice its mass.

Therefore, mass of deuteron, $m = 3.34 \times 10^{-27}$ kg

and its charge, $q = e = 1.6 \times 10^{-19}$ C.

In a cyclotron that has uniform magnetic field B, the angular velocity of circular motion of a particle of charge q, mass m is

$$\omega = \frac{v}{r} = \frac{qB}{m},$$

and the corresponding frequency is

$$f = \frac{\omega}{2\pi} = \frac{qB}{2\pi m}.$$

The final speed of the charged particles is determined by the radius R at which the particles leave the accelerator. The corresponding (nonrelativistic kinetic energy) of the particles is

$$K_{final} = \frac{m{v_f}^2}{2} = \frac{q^2B^2R^2}{2m} = 2\pi^2 m f^2 R^2.$$

The oscillator frequency of the cyclotron is

$f = 12\times10^6$ Hz.

Therefore, the final kinetic energy of the deuterons will be

$$\begin{aligned} K_{final} = 2\pi^2 m f^2 R^2 &= 2\pi^2 \times 3.34\times10^{-27} \times \left(12\times10^6\right)^2 \times \left(0.53\right)^2 \text{ J} \\ &= 2.66\times10^{-12} \text{ J} = 16.6 \text{ MeV}. \end{aligned}$$

The accelerator potential across the dees is 80 kV. Therefore, kinetic energy of the deuterons will increase each time they cross the dee-gap by an amount

$\Delta K = 80\times10^3$ eV.

Therefore, the total number of accelerations that a deuteron will receive during its acceleration in the cyclotron will be

$$n = \frac{K_{final}}{\Delta K} = \frac{16.6\times10^6 \text{ eV}}{80\times10^3 \text{ eV}} ;\ 208.$$

The kinetic energy of the deuteron, after undergoing acceleration each time will change by ΔK. That is it will change from

$K_{initial} = \Delta K,$ to $2\Delta K,\ 3\Delta \mathrm{K}, \ldots n\Delta K.$

And, the speed of the deuteron will change from

$$v_1 = \sqrt{\frac{2\Delta K}{m}}, \text{ to } v_2 = \sqrt{2}v_{1,} \text{to } v_3 = \sqrt{3}v_1 \ldots v_n = \sqrt{n}v_1.$$

With constant speeds deuterons will travel for time

$$\frac{T}{2} = \frac{1}{2f} \text{ s.}$$

Therefore, the total distance a deuteron will travel before leaving the cyclotron will be

$$d = \left(v_1 + v_2 + \ldots + v_n\right) \times \frac{T}{2} = \sum_{i=1}^{n=208} v_i \times \frac{1}{2f} = \frac{v_1}{2f} \sum_{i=1}^{n=208} \sqrt{i}$$

$$= \sqrt{\frac{2\Delta K}{m}} \times \frac{1}{2f} \sum_{i=1}^{n=208} \sqrt{i}$$

Or,

$$\mathrm{d} = \sqrt{\frac{2 \times 80 \times 10^3 \times 1.6 \times 10^{-19}}{3.34 \times 10^{-27}}} \times \frac{1}{24 \times 10^6} \sum_{i=1}^{n=208} \sqrt{i} \text{ m}$$

$$= 0.115 \sum_{i=1}^{n=208} \sqrt{i} \text{ m.}$$

The reader is encouraged to evaluate the sum of square roots of natural numbers ranging from 1 to 208 and see the resultant value….

$$\sum_{i=1}^{n=208} \sqrt{i} = 2005.14,$$

Therefore,

$$d = 2005.14 \times 0.115 \text{ m} = 230.6 \text{ m}.$$