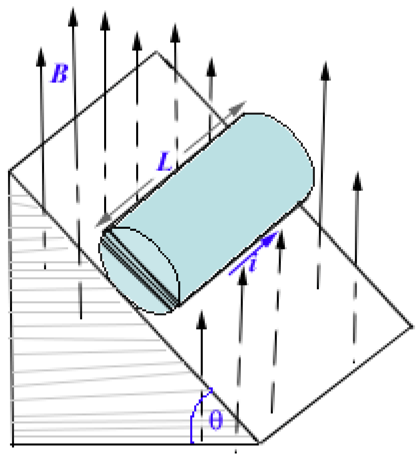
451.

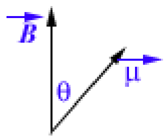
<u>Problem 34.55 (RHK)</u>

In the figure a wooden cylinder with a mass $m = 262 \text{ g}$ *and a length* $L = 12.7 \text{ cm}$, *with* $N = 13$ *turns of wire wrapped around it longitudinally has been shown. The plane of the wire loop contains the axis of the cylinder. We have to find the least current through the loop that will prevent the cylinder from rolling down a plane inclined at angle* θ *to the horizontal, in the presence of a vertical, uniform magnetic field of* 477 mT, *if the plane of the windings is parallel to the inclined plane.*



<u>Solution:</u>

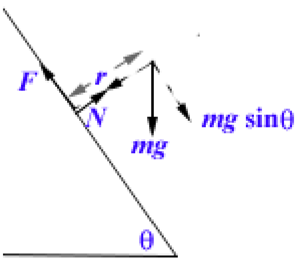
The current carrying loop will behave like a magnetic dipole. As the plane of the windings is parallel to the inclined plane and the direction of the current is as shown in the figure, the magnetic dipole moment $\vec{\mu}$ will be perpendicular to the inclined plane. As the inclined plane makes an angle θ with the horizontal base, the angle between the magnetic field $\vec{B}$ and $\vec{\mu}$ will also be θ.



The magnitude of the torque $\vec{\mu}\times\vec{B}$ will be $\mu B\sin\theta$ and its direction will be counter-clockwise. The torque will try to turn the wire-loop so that $\vec{\mu}$ aligns with $\vec{B}$. The magnitude of $\vec{\mu}$ will be equal to the area of the wire-loop times the number of loops times the current. That is

$\mu = 2rLni.$

The non-magnetic forces on the cylinder will be as shown in the figure.



These forces are the weight mg acting vertically downward, the force of friction $\vec{F}$ acting along the inclined plane and the normal force $\vec{N}$, as shown in the figure. As the cylinder is in equilibrium

$F = mg\sin\theta.$

The force of friction will exert a torque on the cylinder of magnitude

$$\tau_F = rmg\sin\theta.$$

This torque will try to turn the cylinder in the clockwise direction. That is it will cause the cylinder to roll down the inclined plane. We want the cylinder to be in equilibrium. The torque due to the magnetic field and the torque due to the force of friction are required to balance each other.

We, therefore, have the equation

$$2rLniB\sin\theta = rmg\sin\theta,$$

or

$$i = \frac{mg}{2LnB}.$$

Substituting the data of the problem, we find

$$i = \frac{262\times10^{-3}\times9.81}{2\times12.7\times10^{-2}\times13\times477\times10^{-3}}\ \text{A} = 1.63\ \text{A}.$$