449.

Problem 34.51 (RHK)

A length L of wire carries a current i. We have to show that if the wire is formed into a circular coil, the maximum torque in a given magnetic field is developed when the coil has one turn only and the maximum torque has the magnitude

$$\tau = \frac{1}{4\pi} L^2 iB.$$

Solution:

Let us assume that wire of length L is turned into a coil of radius r and n turns. We then have

$$L = 2\pi rn,$$

or

$$r = \frac{L}{2\pi n}.$$

Total area of the n-turns of the coil will be

$$A = n\pi r^2 = \frac{n\pi L^2}{4\pi^2 n^2} = \frac{L^2}{4\pi n}.$$

It is maximum for $n = 1$. A current carrying loop of area A and current i is effectively a magnetic dipole with dipole moment

$$\vec{\mu} = iA\hat{n},$$

where $\hat{n}$ is the unit vector $\perp$ to the plane of the loop. Therefore, the dipole moment of loop will be

$$\vec{\mu} = \frac{iL^2}{4\pi}\hat{n}.$$

The torque on a magnetic dipole of moment $\vec{\mu}$ in magnetic field $\vec{B}$ is

$$\vec{\tau} = \vec{\mu} \times \vec{B}.$$

The maximum value of torque is obtained when $\vec{\mu}$ and $\vec{B}$ are $\perp$ to each other. Its magnitude then is

$$\tau_{\max} = \mu B.$$

For our problem

$$|\vec{\mu}| = \frac{iL^2}{4\pi}.$$

$$\therefore \quad \tau_{\max} = \frac{iL^2 B}{4\pi}.$$