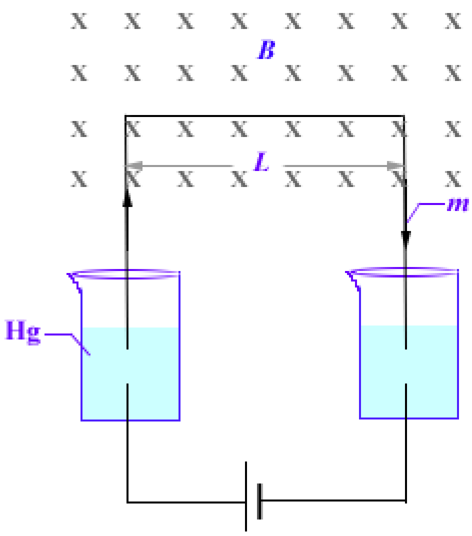
447.

<u>Problem 34.45 (RHK)</u>

A U-shaped wire of mass m and length L is immersed with two ends in mercury (see figure). The wire is in a homogeneous magnetic field, $\vec{B}$*. If a charge, that is, a current pulse,* $q=\int i dt$*, is sent through the wire, the wire will jump up. We have to calculate, from the height that wire reaches, the size of the charge or current pulse, assuming that the time of the current pulse is very small in comparison with the time of flight. We will make use of the fact that impulse of force equals,* $\int F dt$*, which equals* mv*. We will evaluate* q *for* $B = 0.12$ T, $m = 13$ g, $L = 20$ cm, and $h = 3.1$ m.



Solution:

Data for the problem are:

Magnetic field on the wire, $B = 0.12$ T,

Length of the wire (distance between the ends immersed in mercury) in the magnetic field, $L = 0.2$ m,

Mass of the wire, $m = 0.013$ kg,

The height to which the wire jumps, $h = 3.1$ m.

Force on a wire of length L and carrying current i in magnetic field $\vec{B}$ is

$\vec{F} = i\vec{L} \times \vec{B}.$

From the directions of current in the U-shaped wire and that of the magnetic field, as shown in the figure, we note that the force on the vertical sections of the wire will cancel each other, as the current flow in the two sections is in opposite directions. The force on the U-shaped wire is due to the horizontal section of length L and is vertically upward. The magnitude of the force is

$F = iLB.$

The force due to the magnetic field acts for a short time as the circuit will break as soon as the leads immersed in mercury snap out. The action of the force can be treated as an impulse of magnitude

$$\int F dt = BL \int i dt,$$

where

$\int i dt = q$ is the charge that would have been delivered by the battery, seat of emf, to the circuit.

As an impulse force imparts momentum such that

$$\int F dt = LBq = mv,$$

where v is the speed acquired by the wire of mass m.

Height h to which an object of mass m with initial speed v will go up in the gravitational field of the Earth is given by the conservation of energy,

$$\frac{mv^2}{2} = mgh,$$

or

$$v = \sqrt{2gh}.$$

As the wire jumps up to a height of 3.1 m, its initial speed would have been

$$v = \sqrt{2 \times 9.81 \times 3.1} \text{ m s}^{-1} = 7.672 \text{ m s}^{-1}.$$

Therefore,

$$q = \frac{mv}{LB} = \frac{13 \times 10^{-3} \times 7.672}{0.2 \times 0.12} \text{ C} = 4.155 \text{ C} ; \; 4.2 \text{ C}.$$