441.

Problem 34.37 (RHK)

We have to show that, in terms of the Hall electric field E and the current density j, the number of charge carriers per unit volume is given by

$$n = \frac{jB}{eE}.$$

Solution:

The Hall electric field $\vec{E}$ arises so that the net Lorentz force on charge carriers inside a conductor is zero. Let $\vec{v}_d$ be the drift velocity of charge carriers. The magnetic field $\vec{B}$ is so arranged as to have

$\vec{v}_d . \vec{B} = 0.$

This condition implies that

$e\left(\vec{v}_d \times \vec{B} + \vec{E}\right) = 0.$

From this equation, we note that

$\left|\vec{v}_d \times \vec{B}\right| = \left|-\vec{E}\right|.$

As $\vec{v}_d$ and $\vec{B}$ are orthogonal, we have

$v_d B = E.$

The current density j in terms of volume density of charge carriers, n, drift speed, v_d, and charge, e, is

$$j = nv_d e,$$

$$\therefore\ v_d = \frac{j}{ne}.$$

We thus have

$$\frac{E}{B} = \frac{j}{ne},$$

and

$$n = \frac{jB}{eE}.$$