440.

<u>Problem 34.35 (RHK)</u>

Consider a particle of mass m and charge q moving in the xy*-plane under the influence of a uniform magnetic field* $\vec{B}$ *pointing in the* $+z$ *direction. We have to prove that the particle moves in a circular path by solving Newton's equations of motion analytically.*

<u>Solution:</u>

Force on a particle of charge *q*, mass *m*, moving in magnetic field $\vec{B}$ is

$$\vec{F} = q\vec{v} \times \vec{B}.$$

Let the position vector of the particle be $\vec{r}$.

We use the Newton's second law for writing the equation of motion. It is

$$m\frac{d^2\vec{r}(t)}{dt^2} = q\vec{v} \times \vec{B}.$$

It is given that the magnetic field is in the $+z$ direction.

That is

$$\vec{B} = B\hat{k}.$$

As the particle is moving in the xy-plane,

$$\vec{r}(t) = x(t)\hat{i} + y(t)\hat{j}.$$

Substituting this in the equation of motion, we have

$$m\left(\frac{d^2x(t)}{dt^2}\hat{i} + \frac{d^2y(t)}{dt^2}\hat{j}\right) = q\left(v_x\hat{i} + v_y\hat{j}\right)\times B\hat{k}$$

$$= q\left(-v_x\hat{j} + v_y\hat{i}\right)B,$$

or

$$m\frac{d^2x(t)}{dt^2} = qv_yB,$$

and

$$m\frac{d^2y(t)}{dt^2} = -qv_xB.$$

Combining above two equations we get

$$\frac{d}{dt}\left(v_x + i\,v_y\right) = -\frac{iqB}{m}\left(v_x + i\,v_0\right)$$

Its solution is

$$v_x + i\,v_y = v e^{-\frac{iqB}{m}(t+\alpha)}$$

Without loss of generality, we select the solution of this equation as

$$v_y = v\sin\left(\frac{qB}{m}t\right).$$

Therefore,

$$v_x = -v\cos\left(\frac{qB}{m}t\right).$$

Note that

$$v_x^2 + v_y^2 = v^2.$$

That is the charged particle is moving with constant speed and describes a circular motion. Let the radius of the circular orbit be *r*. Integrating v_x and v_y equations, we can write

$$x = -\frac{mv}{qB}\sin\left(\frac{qB}{m}t\right),$$

$$y = -\frac{mv}{qB}\cos\left(\frac{qB}{m}t\right),$$

and

$$r = \frac{mv}{qB}.$$

By plotting $x(t)$ and $y(t)$, we note if q is positive, the particle will move in the clockwise direction.