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Problem 34.32 (RHK)

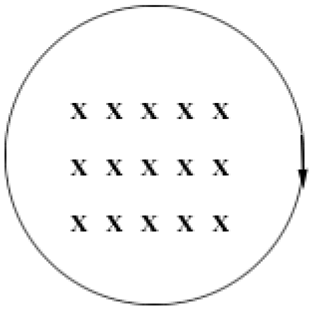
In the Bohr's theory of the hydrogen atom the electron can be thought of as moving in a circular orbit of radius r about the proton. Suppose that such an atom is placed in a magnetic field, with the plane of the orbit at right angles to $\vec{B}$. (a) *If the electron is circulating clockwise, as viewed by an observer sighting along* $\vec{B}$, *we have to answer whether the angular frequency will increase or decrease.* (b) *We have to answer the change in frequency if it is circulating counter-clockwise. Assume that the orbit radius does not change.* (c) *We have to show that the change in frequency of revolution caused by the magnetic field is given approximately by*

$$\Delta \nu = \pm \frac{Be}{4\pi m}.$$

Such frequency shifts were observed by Zeeman in 1896.

Solution:

(a)



Electron has negative charge. Therefore, when it is circulating clockwise as shown in the figure, the centripetal force will increase because of the contribution of the force on the moving electron by the magnetic field. As we assume that the radius of the orbit does not change because of the force due to the magnetic field, the electron will revolve with a faster speed and therefore the angular frequency will increase.

(b)

In the other case when it is circulating in counter-clockwise direction as seen along the direction of the magnetic field, the contribution to the centripetal force on the electron due to the magnetic field will be opposite to that of the Coulomb attraction by the proton, and hence the centripetal force will decrease. Therefore, the angular frequency will decrease.

(c)

Before the hydrogen atom is placed in a magnetic field, in the Bohr's model of atom the centripetal force on the electron is due to the Coulomb' attraction on the electron due to the proton at the centre of the atom. That is

$$\frac{mv_0^2}{r} = \frac{e^2}{4\pi\varepsilon_0 r^2},$$

where v_0 is the orbital speed of the electron and r is the radius of the circular orbit.

When the magnetic field, perpendicular to the plane of the orbit, is applied to the atom the orbital speed will increase or decrease depending on whether the electron is circulating clockwise or anti-clockwise (parts (a) and (b) of the problem). Let the changed speed of the electron be $v_0 \pm \Delta v_0$. The contribution to the centripetal force on the electron due to the magnetic field will be $\pm e v_0 B$.

The changed equation of motion will be

$$\frac{m\left(v_0 \pm \Delta v_0\right)^2}{r} = \frac{e^2}{4\pi\varepsilon_0 r^2} \pm e v_0 B.$$

Retaining terms up to first order in Δv_0, we get

$$\frac{m v_0^2}{r} \pm \frac{2 m v_0 \Delta v_0}{r} = \frac{e^2}{4\pi\varepsilon_0 r^2} \pm e v_0 B,$$

or

$$\pm \frac{2 m v_0 \Delta v_0}{r} = \pm e v_0 B,$$

and

$$\frac{\Delta v_0}{r} = \frac{eB}{2m}.$$

And, the change in frequency of the orbit will be

$$\Delta \nu = \pm \frac{\Delta v_0}{2\pi r} = \pm \frac{eB}{4\pi m}.$$