

438.

Problem 34.31 (RHK)

A 22.5-eV positron (positively charged electron) is projected into a uniform magnetic field $B = 455 \mu\text{T}$ with its velocity vector making an angle of 65.5° with \hat{B} . We have to find (a) the period, (b) the pitch p , and (c) the radius r of the helical path.

Solution:

It is given that the velocity vector makes an angle of 65.5° with the magnetic field vector \hat{B} . Magnitude of the magnetic field vector

$$B = 455 \mu\text{T}.$$

We will first find the magnitude of velocity of a 22.5-eV positron.

$$\frac{1}{2}m_e v^2 = 22.5 \times 1.6 \times 10^{-19} \text{ J},$$

$$\therefore v = \left(\frac{2 \times 22.5 \times 1.6 \times 10^{-19}}{9.11 \times 10^{-31}} \right)^{1/2} \text{ m s}^{-1} = 2.8113 \times 10^6 \text{ m s}^{-1}.$$

We resolve the velocity vector into components, v_p , which is parallel to the field, and v_{\perp} , which is orthogonal to the magnetic field vector \vec{B} .

$$v_p = v \cos 65.5^\circ = 2.8113 \times 10^6 \times \cos 65.5^\circ \text{ m s}^{-1} = 1.1658 \times 10^6 \text{ m s}^{-1}$$

and

$$v_{\perp} = 2.8113 \times 10^6 \times \sin 65.5^\circ \text{ m s}^{-1} = 2.5581 \times 10^6 \text{ m s}^{-1}.$$

The radius of the helical path will be determined by v_{\perp} and the pitch by v_p .

We have

$$\frac{m_e v_{\perp}}{r} = eB,$$

and

$$\therefore \frac{2\pi}{T} = \frac{v_{\perp}}{r} = \frac{eB}{m_e}.$$



The period of the circular orbit is therefore

$$T = \frac{2\pi m_e}{B} = \frac{2\pi \times 9.11 \times 10^{-31}}{1.6 \times 10^{-19} \times 455 \times 10^{-6}} \text{ s} = 7.86 \times 10^{-8} \text{ s} = 78.6 \text{ ns}.$$

The pitch of the helical path

$$p = T \times v_p = 7.86 \times 10^{-8} \times 1.1658 \times 10^6 \text{ m} \\ = 9.16 \text{ cm}.$$

The radius of the helical path

$$r = \frac{m_e v_{\perp}}{eB} = \frac{T}{2\pi} v_{\perp} = \frac{7.86 \times 10^{-8} \times 2.5581 \times 10^6}{2\pi} \text{ m} = 3.2 \text{ cm}.$$