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Problem 34.27 (RHK)

We have to calculate the speed that a proton should have for circling the Earth at the equator, if the Earth's magnetic field is everywhere horizontal there and directed along the longitudinal lines. Relativistic effects have to be taken into account. We can take the magnitude of the Earth's magnetic field to be $41\,\mu\mathrm{T}$ *at the equator. We have to draw the velocity and the magnetic field vectors corresponding to this situation.*

Solution:

Let R_E be the radius of the Earth. The relativistically correct relation for circular orbit, radius R, of a charged particle, charge q, with momentum p, moving in magnetic field, B, perpendicular to the velocity of the particle, is

$$\frac{p}{R} = qB.$$

Therefore, magnitude of the momentum of proton moving around the Earth at the equator has to be

$$p = eBR_E = 1.6\times10^{-19}\times41\times10^{-6}\times6.4\times10^{6}\ \text{kg m s}^{-1}$$
$$= 4.1984\times10^{-17}\,\text{kg m s}^{-1}.$$

In calculating the momentum of the proton, we have used the following data:

$R_E = 6.4\times10^{6}$ m,

and

$B = 41\times10^{-6}$ T.

The relativistic momentum, p, of a particle of rest mass, m, and velocity v is

$$p = \frac{mv}{\sqrt{1-v^2/c^2}},$$

or

$$p^2 = v^2\left(m^2 + p^2/c^2\right),$$

or

$$v = \frac{pc}{\left(p^2 + m^2c^2\right)} = \frac{c}{\left(1+\left(mc/p\right)^2\right)}.$$

Mass of a proton, $m_p = 1.67\times10^{-27}$ kg.

Therefore, the speed of the proton will be

$$v=\frac{c}{\left(1+\left(\frac{1.67\times10^{-27}\times3\times10^{8}}{4.198\times10^{-17}}\right)^{2}\right)^{1/2}}$$

$$=\frac{c}{(1+0.000142)^{1/2}}=0.999929c.$$

The velocity and the magnetic field vectors will have to be oriented as shown in the figure. As the force has to be towards the centre of the Earth, i.e. piercing into the plane of the figure, the velocity of the proton has to be in the direct of east–to–west.