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<u>Problem 34.7 (RHK)</u>

An electron in a uniform magnetic field has a velocity $\vec{v} = \left(40\hat{i} + 35\hat{j}\right)$ km s^{-1}. *It experiences a force* $\vec{F} = \left(-4.2\hat{i} + 4.8\hat{j}\right)$ fN. *If* $B_x = 0$, *we have to calculate the magnetic field.*

<u>Solution:</u>

The force on an electron, charge e, velocity $\vec{v}$, in magnetic field $\vec{B}$ is given by the equation

$$\vec{F} = e\vec{v} \times \vec{B}.$$

Data of the problem are

$$\vec{v} = \left(40\hat{i} + 35\hat{j}\right) \text{ km s}^{-1},$$

$$\vec{F} = \left(-4.2\hat{i} + 4.8\hat{j}\right) \text{ fN} = \left(-4.2\hat{i} + 4.8\hat{j}\right) \times 10^{-15} \text{ N}.$$

As $B_x = 0$, we will write the magnetic field as

$$\vec{B} = \left(B_y\hat{j} + B_z\hat{k}\right).$$

We thus have the vector equation

$$\left(-4.2\hat{i} + 4.8\hat{j}\right) \times 10^{-15} \text{ N} = e\left(40\hat{i} + 35\hat{j}\right) \text{ km s}^{-1} \times \left(B_y\hat{j} + B_z\hat{k}\right). \quad \text{(A)}$$

We will use the properties of the cross- product

$\hat{i} \times \hat{j} = \hat{k};\ \hat{j} \times \hat{k} = \hat{i};\ \hat{k} \times \hat{i} = \hat{j};$

and

$\vec{a} \times \vec{b} = -\vec{b} \times \vec{a};$ and $\vec{a} \times \vec{a} = 0,$

and rewrite equation (A) as

$$\left(-4.2\hat{i} + 4.8\hat{j}\right) \times 10^{-15}\ \mathrm{N} = \left(40B_y\hat{k} - 40B_z\hat{j} + 35\hat{i}B_z\right)e\ \mathrm{km\ s}^{-1}.$$

$\therefore\ B_y = 0,$

and

$$B_z = \frac{4.2 \times 10^{-15}}{1.6 \times 10^{-19} \times 35} \times \frac{\mathrm{N}}{\mathrm{C\ km\ s^{-1}}} = 7.5 \times 10^2\ \mathrm{T}.$$

It may be noted that the $\hat{j}$-equation is consistent with the $\hat{i}$-equation, as it also gives

$$B_z = \frac{4.8 \times 10^{-15}}{1.6 \times 40 \times 10^{-19}} \times \frac{\mathrm{N}}{\mathrm{C\ km\ s^{-1}}} = 7.5 \times 10^2\ \mathrm{T}.$$

Therefore, the magnetic field is

$\vec{B} = 0.75\hat{k}\ \mathrm{kT}.$