421.

Problem 33.52 (RHK)

A controller on an electronic arcade game consists of a variable resistor connected across the plates of a 220-nF *capacitor. The capacitor is charged to* 5.00 V, *then discharged through the resistor. The time for the potential difference to decrease to* 800 mV *is measured by an internal clock. We have to find the range of the resistance of the resistor, if the range of discharge times that can be handled is from* 10.0 μs *to* 6.0 ms.

Solution:

Capacitance of the capacitor is

$C = 220 \times 10^{-9}$ F.

As equation for discharge of a capacitor in a *RC* circuit is

$q(t) = q_0 e^{-t/RC}$.

Therefore, variation of voltage across the capacitor as it discharges with time will be given by the equation

$V(t) = V_0 e^{-t/RC}$.

It is given that an internal clock measures time, *t*, for the voltage of the capacitor charged to 5.00 V to drop to 800 mV. Therefore, relation between *t* and the resistance *R* is

$$-\frac{t}{RC} = \ln\left(\frac{800\times10^{-3}}{5}\right)$$

$$= \ln(0.16) = -1.8326,$$

or

$$t = 1.8326\times220\times10^{-9}R\ \left(\Omega^{-1}\text{s}\right) = 4.03\times10^{-7}R\ \left(\Omega^{-1}\text{s}\right).$$

And

$$R = \frac{t(\text{s})}{4.03\times10^{-7}}\ (\Omega).$$

(a)

For $t = 10.0\ \mu\text{s} = 1.0\times10^{-5}\ \text{s}$,

resistance in the circuit has to be

$$R = \frac{10^{-5}}{4.03\times10^{-7}}\ (\Omega) = 24.8\ \Omega.$$

(b)

For $t = 6.0\ \text{ms} = 6.0\times10^{-3}\ \text{s}$,

resistance in the circuit has to be

$$R = \frac{6\times10^{-3}}{4.03\times10^{-7}}\ (\Omega) = 14{,}888.0\ \Omega = 14.88\ \text{k}\Omega.$$

Therefore, the range of the resistance of the resistor has to be from 24.8 Ω to 14.9 kΩ.