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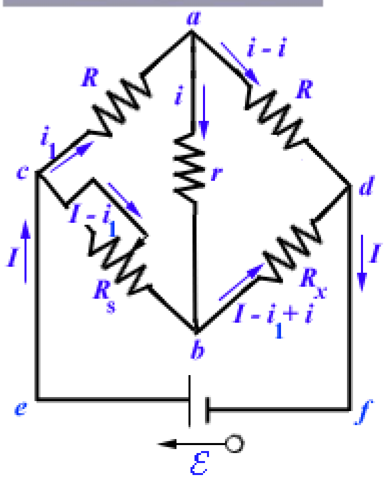
Problem 33.47 (RHK)

If points a and b in the circuit shown in the figure are connected by a wire of resistance r, we have to show that the current in the wire is

$$i = \frac{\mathrm{E}\left(R_s - R_x\right)}{\left(R + 2r\right)\left(R_s + R_x\right) + 2R_s R_x},$$

where E *is the emf of the battery. Assume that* R_1 *and* R_2 *are equal* $\left(R_1 = R_2 = R\right)$. *We have to show that this formula is consistent with the Wheatstone bridge condition.*



Solution:

It is a problem of multi-loop circuit. We have marked currents in different branches of the circuit requiring that at each junction the total current that is entering is equal to the total current that is leaving.

Applying Kirchoff's laws to the loops *ecadfe*, *cabc*, *adba*, we write the following equations:

$$-i_1R-(i_1-i)R+\mathrm{E}=0,$$

or

$$2i_1R-iR=\mathrm{E}\ . \qquad (\mathrm{A})$$

$$-i_1R-ir+(I-i_1)R_\mathrm{s}=0,$$

or

$$-i_1(R+R_\mathrm{s})+IR_\mathrm{s}-ir=0\quad . \quad (\mathrm{B})$$

And

$$-(i_1-i)R+(I-i_1+i)R_x+ir=0,$$

or

$$-i_1(R+R_x)+IR_x+i(R+R_x+r)=0\ .\ (\mathrm{C})$$

We have three linear equations, (A), (B) and (C), in three unknowns, $i,\ i_1$ and I. We solve them and find that the current in the wire joining points *a* and *b* is

$$i=\frac{\mathrm{E}(R_\mathrm{s}-R_x)}{(R+2r)(R_\mathrm{s}+R_x)+2R_\mathrm{s}R_x}.$$

Note, if $R_\mathrm{s}=R_x$, the condition for Wheatstone bridge, $i=0$. That is points *a* and *b* are equipotential and there is no flow of current in the wire that joins them.