407.

Problem 33.15 (RHK)

A battery of emf $\mathcal{E} = 2.0\ \text{V}$ *and internal resistance* $r = 0.50\ \Omega$ *is driving a motor. The motor is lifting a* 2.0-N *object at constant speed* $v = 0.50\ \text{m s}^{-1}$. *Assuming no power losses, we have to find* (a) *the current i in the circuit and* (b) *the potential difference V across the terminals of the motor.* (c) *We have to discuss the fact that there are two solutions to the problem.*

Solution:

(a)

The rate at which electrical energy in the motor is being used in gaining gravitational potential energy is

$W = mgv = 2.0 \times 0.50\ \text{W} = 1.0\ \text{W}.$

Let the current in the circuit be *i* A. The power supplied by the battery is

$P = iV = 2.0i\ \text{W}.$

It is equal to the Joule heat in the internal resistance of the battery plus the power spent at the motor in lifting the object. That is

$P = i^2 r + W = 0.5 i^2 + 1.0.$

We thus have the following equation for the current

$2.0 i = 0.5 i^2 + 1.0,$

or

$i^2 - 4i + 2 = 0.$

Roots of this quadratic equation are

$$i = \frac{4 \pm \sqrt{16 - 8}}{2} = \frac{4 \pm 2.828}{2};$$

$i = 3.41$ A, 0.586 A.

(b) and (c)

There are therefore two solutions for this problem.

When the current in the circuit is 3.41 A the potential difference across the terminals of the motor will be

$\left(2.0 - 3.41 \times 0.5\right)$ V $= 0.29$ V.

When the current in the circuit is 0.586 A the potential difference across the terminals of the motor will be

$\left(2.0 - 0.586 \times 0.5\right)$ V $= 1.71$ V.