398.

Problem 32.61 (RHK)

A potential difference V is applied to a wire of cross-sectional area A, length L, and conductivity σ. *We want to change the applied potential difference and draw out the power so the power dissipated is increased by a factor of* 30 *and the current is increased by a factor of* 4. *We have to find the new values of* (a) *the length and* (b) *the cross-sectional area.*

Solution:

Let the original resistance of the wire be *R*. When a potential difference V is applied to the ends of the wire the current in it will be

$$i = \frac{V}{R},$$

and the power dissipated will be given by

$$P = i^2 R.$$

Wire is stretched and potential difference is applied so that the current through the wire becomes i', such that

$$i' = 4i.$$

Let the changed resistance R' of the wire that ensures that when the current is i' the power dissipated becomes

$$P' = i'^2 R' = 30 i^2 R,$$

and

$$16 i^2 R' = 30 i^2 R.$$

This gives

$$R' = \frac{15}{8} R.$$

As the wire is stretched its volume remains unchanged. Therefore,

$$A'L' = AL.$$

Resistance of a wire of cross-sectional area A, length L, and conductivity of the material σ is given by

$$R = \frac{L}{\sigma A}.$$

Therefore, we have the relation

$$\frac{L'}{\sigma A'} = \frac{15}{8} \times \frac{L}{\sigma A},$$

or

$$L' = \frac{15}{8}\left(\frac{A'}{A}\right) L.$$

Using the result

$$A'L' = AL,$$

we have

$$A' = \sqrt{\frac{8}{15}}A = 0.730A,$$

and

$$L' = \sqrt{\frac{15}{8}}L = 1.37L.$$