

398.

**Problem 32.61 (RHK)**

*A potential difference  $V$  is applied to a wire of cross-sectional area  $A$ , length  $L$ , and conductivity  $\sigma$ . We want to change the applied potential difference and draw out the power so the power dissipated is increased by a factor of 30 and the current is increased by a factor of 4. We have to find the new values of (a) the length and (b) the cross-sectional area.*



**Solution:**

Let the original resistance of the wire be  $R$ . When a potential difference  $V$  is applied to the ends of the wire the current in it will be

$$i = \frac{V}{R},$$

and the power dissipated will be given by

$$P = i^2 R.$$

Wire is stretched and potential difference is applied so that the current through the wire becomes  $i'$ , such that

$$i' = 4i.$$

Let the changed resistance  $R'$  of the wire that ensures that when the current is  $i'$  the power dissipated becomes

$$P' = i'^2 R' = 30i^2 R,$$

and

$$16i^2 R' = 30i^2 R.$$

This gives

$$R' = \frac{15}{8} R.$$

As the wire is stretched its volume remains unchanged.

Therefore,

$$A'L' = AL.$$

Resistance of a wire of cross-sectional area  $A$ , length  $L$ , and conductivity of the material  $\sigma$  is given by

$$R = \frac{L}{\sigma A}.$$

Therefore, we have the relation

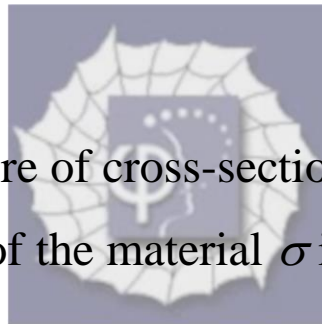
$$\frac{L'}{\sigma A'} = \frac{15}{8} \times \frac{L}{\sigma A},$$

or

$$L' = \frac{15}{8} \left( \frac{A'}{A} \right) L.$$

Using the result

$$A'L' = AL,$$



we have

$$A' = \sqrt{\frac{8}{15}}A = 0.730A,$$

and

$$L' = \sqrt{\frac{15}{8}}L = 1.37L.$$

