397.

Problem 32.57 (RHK)

Two isolated conducting spheres, each of radius 14.0 cm, *are charged to potentials of* 240 *and* 440 V *and are then connected by a fine wire. We have to calculate the internal energy developed in the wire.*

Solution:

Potential difference with respect to infinity of a conducting sphere of radius r and total charge q on its surface is given by

$$V = \frac{q}{4\pi\varepsilon_0 r}.$$

Therefore, the capacitance of a conductor of radius r is

$$C = \frac{q}{V} = 4\pi\varepsilon_0 r.$$

The other fact that we will use in answering this problem is that energy associated with a condenser of capacitance C and charge q is

$$U = \frac{q^2}{2C} = \frac{1}{2}CV^2.$$

Therefore, charge on the conductor at 240 V potential will be

$$q_1 = CV = 4\pi\varepsilon_0 rV = \frac{14.0\times10^{-2}\times240}{8.99\times10^{9}}\ \text{C} = 3.737\times10^{-9}\ \text{C}.$$

Similarly, the charge on the conductor at 440 V potential will be

$$q_2 = \frac{14\times10^{-2}\times440}{8.99\times10^{9}}\ \text{C} = 6.852\times10^{-9}\ \text{C}.$$

After the two charged conductors have been connected by a fine wire, they will have equal charge as they are of equal radius. So the charge on each conductor after they have been connected will be

$$q = \frac{1}{2}\left(q_1+q_2\right) = \frac{1}{2}\left(3.737+6.852\right)\times10^{-9}\ \text{C} = 5.294\times10^{-9}\ \text{C}.$$

The common potential on the two conductors will be

$$V = \frac{q}{C} = \frac{5.294\times10^{-9}}{4\pi\varepsilon_0 r} = \frac{8.99\times10^{9}\times5.294\times10^{-9}}{14.0\times10^{-2}}\ \text{V}$$

$$= 339.9\ \text{V}.$$

The change in energy of the two conductors after they are connected by a fine wire will be

$$W = U_i - U_f = \frac{1}{2}C\left(V_1^2 + V_2^2 - 2V^2\right)$$

$$= \frac{14.0\times10^{-2}}{2\times8.99\times10^{9}}\left(240^2 + 440^2 - 2\times340^2\right)\ \text{J}$$

$$= 1.559\times10^{-7}\ \text{J}$$

$$= 155.9\ \text{nJ}.$$

From the conservation of energy, we note that the energy developed in the wire will be 156 nJ.