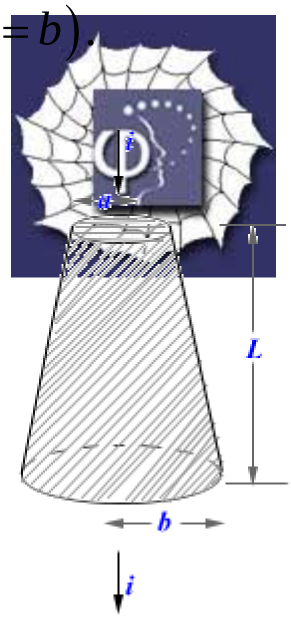
393.

<u>Problem 32.40 (RHK)</u>

A resistor is in the shape of a truncated right circular cone. The end radii are a and b, and the altitude is L. If the taper is small, we may assume that the current density is constant across any cross section. (a) *We have to calculate the resistance of this object.* (b) *We have to show that our answer reduces to* $\rho L/A$ *for the special case of zero taper* $(a = b)$.



<u>Solution:</u>

(a)

We consider the resistor as shown in the figure. It is in the shape of a truncated right circular cone of end radii a and b. We first determine the equation of the taper.

Assuming that it is linear, we describe it by the equation

$$r = mx + c.$$

We determine its constants using the boundary conditions

$$x = 0,\ r = a,$$
$$x = L,\ r = b.$$

We find the constants and fix the line by the equation

$$r = \frac{(b-a)}{L}x + a.$$

We assume that the current density is uniform across any cross section of the resistor. Contribution to the resistance from the section of the cone of width dx at a distance x from the top end will be

$$\Delta R = \frac{\rho dx}{\pi (mx + c)^2},$$

$$m = \frac{(b-a)}{L},\ \text{and } c = a.$$

Therefore, the resistance of the right-circular cone will be

$$R = \int_0^L \frac{dx}{\pi\left(mx+c\right)^2} = -\frac{\rho}{\pi m}\left[\frac{1}{\left(mx+c\right)}\right]_0^L = \frac{\rho}{\pi m}\left(\frac{1}{c}-\frac{1}{Lm+c}\right)$$

$$= \frac{\rho}{\pi\left(\frac{b-a}{L}\right)}\left(\frac{1}{a}-\frac{1}{b}\right)$$

$$= \frac{\rho L}{\pi ab}.$$

(b)

If $a = b$,

$$R = \frac{\rho L}{\pi a^2} = \frac{\rho L}{A}.$$