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Problem 32.35 (RHK)

Copper and aluminium are being considered for a high-voltage transmission line that must carry a current of 62.3 A. *The resistance per unit length is to be* $0.152\ \Omega\ \text{km}^{-1}$. *We have to compute for each choice of material* (a) *the current density and* (b) *the mass of* 1.00 m *of cable. The densities of copper and aluminium are* 8960 *and* 2700 kg m^{-3}, *respectively.*

Solution:

The mass densities of copper and aluminium are

$\rho_{m,Cu} = 8960\ \text{kg m}^{-3}$,

and

$\rho_{m,Al} = 2700\ \text{kg m}^{-3}$.

And electrical resistivities of these materials are

$\rho_{c,Cu} = 1.69\times10^{-8}\ (\Omega\ \text{m})$,

and

$\rho_{c,Al} = 2.75\times10^{-8}\ (\Omega\ \text{m})$.

(a)

We will first find the cross-sectional areas of the copper and aluminium wires, having resistance per unit length of $0.152\ \Omega\ \text{km}^{-1}$. We will use the result that resistance, R, of a wire of length, l, resistivity, ρ_c, and cross-sectional area, A, is given by

$$R = \frac{\rho_c l}{A}.$$

Therefore, for copper wire

$$A_{cu} = \frac{\rho_{c,Cu} \times l}{R} = \frac{1.69 \times 10^{-8} \times 1000}{0.152}\ \text{m}^2 = 1.118 \times 10^{-4}\ \text{m}^2$$
$$= 1.118\ \text{cm}^2.$$

And, similarly for aluminium wire

$$A_{Al} = \frac{\rho_{c,Al} \times l}{R} = \frac{2.75 \times 10^{-8} \times 10^3}{0.152}\ \text{m}^2 = 1.809 \times 10^{-4}\ \text{m}^2$$
$$= 1.809\ \text{cm}^2.$$

The transmission line is to carry current of 62.3 A.

Therefore, the current density in the copper cable will be

$$j_{Cu} = \frac{62.3}{1.118 \times 10^{-4}}\ \text{A m}^{-2} = 5.57 \times 10^5\ \text{A m}^{-2} = 55.7\ \text{A cm}^2,$$

and the current density in the aluminium cable will be

$$j_{Al} = \frac{62.3}{1.809 \times 10^{-4}}\ \text{A m}^{-2} = 3.44 \times 10^5\ \text{A m}^{-2} = 34.4\ \text{A cm}^{-2}.$$

(b)

Mass of 1-m of copper cable will be

$$m_{Cu} = \rho_{m,Cu} \times A_{Cu} = 8960 \times 1.118 \times 10^{-4} \text{ kg} = 1.001 \text{ kg},$$

and the mass of 1-m of aluminium cable will be

$$m_{Al} = \rho_{m,Al} A_{Al} = 2700 \times 1.809 \times 10^{-4} \text{ kg} = 0.488 \text{ kg}.$$