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Problem 32.16 (RHK)

The current density across a cylindrical conductor of radius R varies according to the equation

$$j = j_0\left(1 - r/R\right),$$

where r is the distance from the axis. Thus the current density is a maximum j_0 *at the axis* $r = 0$ *and decreases linearly to zero at the surface* $r = R$. (a) *We have to calculate the current in terms of* j_0 *and the conductor's cross-sectional area* $A = \pi r^2$. (b) *Suppose that, instead, the current density is a maximum* j_0 *at the surface and decreases linearly to zero at the axis, so that*

$$j = j_0\, r/R.$$

We have to calculate the current for this current density. We have to answer why the results (a) *and* (b) *are different.*

Solution:

The current density across a cylindrical conductor of radius R varies according to the equation

$$j = j_0\left(1 - r/R\right),$$

where r is the distance from the axis. Therefore, the current flowing through the wire can be obtained by integrating the current density over the cross-sectional area. That is

$$i = \int_0^R 2\pi r j\left(r\right)dr = \int_0^R 2\pi r j_0\left(1 - \frac{r}{R}\right)dr = \frac{\pi j_0 R^2}{3} = \frac{1}{3} j_0 A.$$

(b)

We now suppose that, instead, the current density is a maximum j_0 at the surface and decreases linearly to zero, so that

$$j = j_0\, r/R.$$

The current flowing through the wire in this situation will be

$$i' = \int_0^R 2\pi r j\left(r\right)dr = 2\pi\, j_0 \int_0^R \frac{r^2}{R}dr = 2\pi\, j_0\, \frac{R^2}{3} = \frac{2}{3} j_0 A.$$

We note that

$$i' = 2i.$$

It should be so. Because when the current density is more at the surface more charge will flow through it compared to when the current density is more near the axis of the wire.