384.

Problem 31.61 (RHK)

A parallel-plate capacitor has plates of area $0.118\ \text{m}^2$ *and a separation of* $1.22\ \text{cm}$. *A battery charges the plates to a potential difference of* $120\ \text{V}$ *and is then disconnected. A dielectric slab of thickness* $4.30\ \text{mm}$ *and dielectric constant* 4.80 *is then placed symmetrically between the plates.* (a) *We have to find the capacitance before the slab is inserted;* (b) *the capacitance with the slab in place;* (c) *the free charge q before and after the slab is inserted;* (d) *the electric field between the plates and the dielectric;* (e) *the electric field in the dielectric;* (f) *the potential difference across the plates when the dielectric slab is in place; and* (g) *the external work involved in the process of inserting the slab.*

Solution:

Data of the problem are:

plate area, $A = 0.118\ \text{m}^2$,

plate separation, $d = 1.22 \times 10^{-2}\ \text{m}$,

thickness of the dielectric slab, $b = 4.30 \times 10^{-3}\ \text{m}$, dielectric constant, $\kappa_e = 4.80$,

EMF of the battery, $V_{battery} = 120$ V.

(a)

Capacitance of the parallel-plate condenser before the dielectric slab is inserted will be

$$C_{air} = \frac{\varepsilon_0 A}{d} = \frac{8.85\times10^{-12}\times0.118}{1.22\times10^{-2}}\ \text{F} = 0.856\times10^{-10}\ \text{F} = 85.6\ \text{pF}.$$

(b)

Capacitance with the slab in place is given by the relation (see Problem 383)

$$C_{dielectric} = \frac{\varepsilon_0 \kappa_e A}{\kappa_e d - b(\kappa_e - 1)}.$$

Therefore, the capacitance of the condenser after the dielectric slab is inserted between the plates will be

$$C_{dielectric} = \frac{8.85\times10^{-12}\times4.80\times0.118}{(4.80\times1.22\times10^{-2}) - 4.30\times10^{-3}(4.80-1)}\ \text{F}$$

$$= 1.18\times10^{-10}\ \text{F} = 118\ \text{pF}.$$

We will also calculate the capacitance of the condenser after the dielectric slab has been inserted directly from the potential difference across the plates and the magnitude of free charge.

(c)

The free charge q on the plates before the slab is inserted is determined by the EMF of the charging the condenser and the capacitance, C_{air}. Therefore,

$$q = C_{air}V_{battery} = 85.6\times10^{-12}\times120\ \text{C} = 10.27\times10^{-9}\ \text{C} = 10.3\ \text{nC}.$$

After the battery has been disconnected and even after the dielectric slab is inserted, free charge q on the capacitor plates will remain unchanged and will be 10.3 nC.

(d)

Electric field in the space between the plates and the dielectric is given by

$$E_{air} = \frac{q}{\varepsilon_0 A},$$

and will be

$$E_{air} = \frac{10.3\times10^{-9}}{8.85\times10^{-12}\times0.118}\ \text{V m}^{-1} = 9.86\ \text{kV m}^{-1}.$$

(e)

Electric field inside the dielectric will be

$$E_{dielectric} = \frac{E_{air}}{\kappa_e} = \frac{9.86}{4.80}\ \text{kV m}^{-1}$$

$$= 2.05\ \text{kV m}^{-1}.$$

(f)

The potential difference across the plates with the dielectric slab inserted will be given by

$$V' = E_{air}\left(d-b\right) + E_{dielectric}b$$

$$= \left(9.86\times10^{3}\left(12.2-4.3\right)\times10^{-3} + 2.05\times10^{3}\times4.30\times10^{-3}\right)\ \text{V}$$

$$= \left(77.89+8.81\right)\ \text{V} = 86.7\ \text{V}.$$

The capacitance of the parallel-plate condenser can now be calculated as it is the ratio of free charge on the plates and the potential across the plates. We have

$$C_{dielectric} = \frac{q}{V'} = \frac{10.27\times10^{-9}}{86.7}\ \text{F=118 pF.}$$

(g)

Electric energy of the capacitor before the slab is inserted is

$$U_{air} = \frac{1}{2}C_{air}V_{battery}^{2} = \frac{85.6\times10^{-12}\times120^{2}}{2}\ \text{J} = 0.616\times10^{-6}\ \text{J}.$$

Electric energy of the capacitor after the dielectric slab is inserted will be

$$U_{dielectric} = \frac{1}{2}C_{dielectric}V'^{2} = \frac{118.6\times10^{-12}\times86.7^{2}}{2}\ \text{J} = 0.446\times10^{-6}\ \text{J}.$$

Therefore, the work done in inserting the slab will be

$$W = -\left(U_{dielectric} - U_{air}\right) = \left(0.616-0.446\right)\times10^{-6}\ \text{J} = 0.17\times10^{-6}\ \text{J}$$

$$= 170\ \text{nJ}.$$