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Problem 31.62 (RHK)

A dielectric slab of thickness b is inserted between the plates of a parallel-plate capacitor of plate separation d. We have to show that the capacitance is given by

$$C = \frac{\kappa_e \varepsilon_0 A}{\kappa_e d - b(\kappa_e - 1)}.$$

Solution:

Let the free charge on the parallel-plate capacitor be q. The electric field between the plates of the capacitor outside the dielectric is given by

$$E = \frac{q}{\varepsilon_0 A},$$

and the electric field inside the dielectric slab is given by

$$E_{dielectric} = \frac{q}{\varepsilon_0 \kappa_e A}.$$

Therefore, the potential difference between the plates will be

$$V = E(d - b) + E_{dielectric} b = \frac{q}{\varepsilon_0 A}\left(d - b + \frac{b}{\kappa_e}\right)$$

$$= \frac{q}{\varepsilon_0 \kappa_e A}\left(\kappa_e d - \kappa_e b + b\right).$$

From the definition of capacitance

$$C = \frac{q}{V},$$

we note that

$$C = \frac{\kappa_e \varepsilon_0 A}{\kappa_e d - b(\kappa_e - 1)}.$$