382.

Problem 31.57 (RHK)

A parallel-plate capacitor has a capacitance of 112 pF, *a plate area of* 96.5 cm^2, *and a mica dielectric* ($\kappa_e = 5.40$). *At a* 55.0-V *potential difference, we have to calculate* (a) *the electric field strength in the mica,* (b) *the magnitude of the free charge on the plates, and* (c) *the magnitude of the induced surface charge*.

Solution:

(a)

In a parallel-plate capacitor electric field, *E*, is constant and is related to the potential difference between the plates, *V*, and the plate separation *d*, by the relation

$V = Ed$.

Capacitance of a parallel-plate capacitor of plate area A and with dielectric, κ_e, filling the space between its plates is

$$C = \frac{\varepsilon_0 \kappa_e A}{d}.$$

The data of the problem are

$C = 112 \text{ pf} = 112 \times 10^{-12} \text{ F},$

$A = 96.5 \text{ cm}^2 = 96.5 \times 10^{-4} \text{ m}^2,$

and

$\kappa_e = 5.40.$

Therefore, separation between the plates is

$$d = \frac{\varepsilon_0 \kappa_e A}{C} = \frac{8.85 \times 10^{-12} \times 5.40 \times 96.5 \times 10^{-4}}{112 \times 10^{-12}} \text{ m} = 41.18 \times 10^{-4} \text{ m}$$
$$= 4.118 \text{ mm}.$$

The electric field strength in the mica will be

$$E = \frac{55}{41.18 \times 10^{-4}} \text{ V m}^{-1} = 13.36 \text{ kV m}^{-1}.$$

(b)

Let the free charge on the capacitor be q. We have

$$E = \frac{q}{\varepsilon_0 \kappa_e A}.$$

Therefore,

$$q = \varepsilon_0 \kappa_e A E = 8.85 \times 10^{-12} \times 5.40 \times 96.5 \times 10^{-4} \times 13.36 \times 10^3 \text{ C}$$
$$= 6.16 \times 10^{-9} \text{ C} = 6.16 \text{ nC}.$$

(c)

The induced surface charge on the plates can be calculated by applying Gauss' law. We have

$\varepsilon_0 E A = q - q_{in},$

and we know

$$\varepsilon_0 EA = \frac{q}{\kappa_e}.$$

Therefore,

$$q_{in} = q\left(1 - \frac{1}{\kappa_e}\right) = 6.16\left(1 - \frac{1}{5.40}\right)\ \text{nC} = 5.02\ \text{nC}.$$