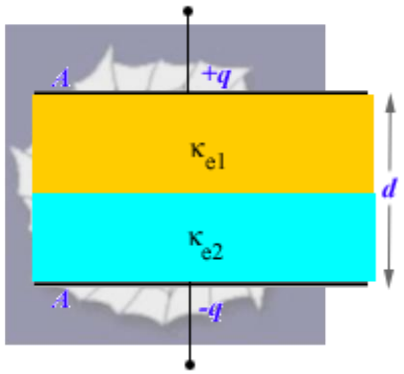
381.

Problem 31.55 (RHK)

A parallel-plate capacitor is filled with two dielectrics as shown in the figure. We have to show that the capacitance is given by

$$C = \frac{2\varepsilon_0 A}{d}\left(\frac{\kappa_{e1}\kappa_{e2}}{\kappa_{e1}+\kappa_{e2}}\right).$$



Solution:

Let q be the free charge on the capacitor. Let the area of the plates be A and the separation between the plates be d. Space between the plates has been filled with dielectrics with dielectric constants κ_{e1} and κ_{e2} as shown in the figure.

The electric field inside the dielectric with dielectric constant κ_{e1} will be

$$E_1 = \frac{q}{\varepsilon_0 \kappa_{e1} A},$$

and the electric field inside the dielectric with dielectric constant κ_{e2} will be

$$E_2 = \frac{q}{\varepsilon_0 \kappa_{e2} A}.$$

Therefore, the potential difference between the plates will be

$$V = \left(E_1 + E_2\right) \times \frac{d}{2} = \frac{qd}{2\varepsilon_0 A}\left(\frac{1}{\kappa_{e1}} + \frac{1}{\kappa_{e2}}\right) = \frac{qd}{2\varepsilon_0 A}\frac{\left(\kappa_{e1} + \kappa_{e2}\right)}{\kappa_{e1}\kappa_{e2}}.$$

As

$$C = \frac{q}{V},$$

we have

$$C = \frac{2\varepsilon_0 A}{d}\left(\frac{\kappa_{e1}\kappa_{e2}}{\kappa_{e1} + \kappa_{e2}}\right).$$