

379.

Problem 31.53 (RHK)

A cylindrical ionisation chamber has a central wire anode of radius 0.180 mm radius and a coaxial cylindrical cathode of radius 11.0 mm. It is filled with a gas whose dielectric strength is 2.20 MV m^{-1} . We have to find the largest potential difference between anode and cathode if gas is to avoid electric breakdown before radiation penetrates the chamber window.



Solution:

Electric field in space filled with gas of dielectric constant κ outside a wire of length L carrying charge q is

$$E(r) = \frac{q}{2\pi r L \epsilon_0 \kappa}.$$

The field will be strongest at the surface of the wire, which is the anode. The radius of the inner wire is 0.180 mm.

The dielectric strength is the largest electric field which can withstand electric breakdown. It is given that that for

the gas filled in the ionisation chamber the dielectric strength is 2.20 MV m^{-1} .

Therefore,

$$\frac{q}{2\pi L\epsilon_0\kappa \times 0.180 \times 10^{-3}} = 2.2 \times 10^6 \text{ V m}^{-1},$$

and we have

$$\frac{q}{2\pi L\epsilon_0\kappa} = 2.2 \times 10^6 \times 0.180 \times 10^{-3} = 0.396 \times 10^3.$$

We now know that the largest electric field between the anode and the cathode of the ionisation chamber which avoids electric breakdown will be

$$E(r) = \frac{0.396 \times 10^3}{r} \text{ V m}^{-1}.$$

Therefore, the largest potential difference that can be applied between the anode and the cathode of the cylindrical ionisation chamber having inner radius 0.180 mm and outer radius 11.0 mm will be

$$V_{\max} = \int_{0.180 \times 10^{-3}}^{11.0 \times 10^{-3}} \frac{0.396 \times 10^3}{r} dr \text{ V} = 0.396 \times 10^3 \ln\left(\frac{11.0}{0.180}\right) \text{ V}$$

$$= 1.63 \times 10^3 \text{ V} = 1.63 \text{ kV}.$$