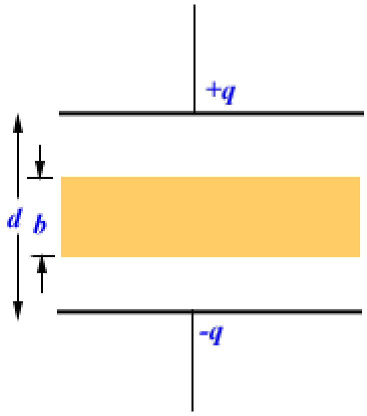
378.

Problem 31.51 (RHK)

A slab of copper of thickness b is thrust into a parallel-plate capacitor as shown in the figure. (a) *We have to find the capacitance after the slab has been introduced.* (b) *If a charge q is maintained on the plates, we have to find the stored energy before to that after the slab is inserted.* (c) *We have to calculate the amount of work that is done when the slab is inserted. We have to answer whether the slab is pulled in or do we have to push it in.*



Solution:

(a)

Let the area of the plates be *A* and separation between the plates be *d*. The capacitance of the parallel-plate capacitor before a slab of copper of thickness *b* is inserted into it will be

$$C = \frac{\varepsilon_0 A}{d}.$$

Let the charge on the plates be $+q$ and $-q$. Electric field in the space between the plates will be

$$E = \frac{q}{\varepsilon_0 A}.$$

We calculate next the capacitance after the copper slab has been inserted between the plates as shown in the figure. As copper is a conducting metal the electric field within the copper slab will be zero. Therefore, the potential difference between the plates after the copper slab has been inserted will be

$$V = E \times (d - b) = \frac{q}{\varepsilon_0 A} \times (d - b).$$

Therefore, the capacitance of the parallel-plate capacitor after the copper slab has been inserted will change to

$$C' = \frac{q}{V} = \frac{\varepsilon_0 A}{(d - b)}.$$

(b)

Stored electric energy in a charged capacitor is given by

$$U = \frac{q^2}{2C}.$$

Therefore, the initial stored energy will be

$$U_i = \frac{q^2}{2C} = \frac{q^2 d}{2\varepsilon_0 A},$$

and the stored energy after the copper slab is inserted will be

$$U_f = \frac{q^2}{2C'} = \frac{q^2 (d-b)}{2\varepsilon_0 A}.$$

The change in energy of the capacitor when the copper slab is inserted between the plates will be

$$U_f - U_i = \frac{q^2 (d-b)}{2\varepsilon_0 A} - \frac{q^2 d}{2\varepsilon_0 A} = -\frac{q^2 b}{2\varepsilon_0 A}.$$

Therefore, the amount of work done on the slab will be

$$W = U_f - U_i = -\frac{q^2 b}{2\varepsilon_0 A}.$$

As W is negative the work is done on the slab by the electric field, therefore, the copper slab will be pulled-in.