375.

Problem 31.42 (RHK)

A soap bubble of radius R_0 *is slowly given a charge q. Because of mutual repulsion of the surface charge, the radius increases slightly to R. The air pressure inside the bubble drops, because of the expansion, to* $p\left(V_0/V\right)$, *where p is the atmospheric pressure,* V_0 *is the initial volume, and V is the final volume. We have to show that*

$$q^2 = 32\pi^2\varepsilon_0 pR\left(R^3 - R_0^3\right).$$

Solution:

Radius of the uncharged soap bubble is R_0. Let the atmospheric pressure be *p*. We are going to neglect the effect of surface tension in this problem. The initial air pressure inside the soap bubble will also be *p*. When the soap bubble is given a charge *q*, electric field on its surface will be

$$E\left(R\right) = \frac{q}{4\pi\varepsilon_0 R^2}.$$

Because of the charge on the soap bubble in addition to the pressure due to air there will be electrostatic pressure. Therefore, for equilibrium pressure inside will decrease such that the air pressure and the electrostatic pressure equal the atmospheric pressure. That is

$$p' + \frac{1}{2}\varepsilon_0 E^2 = p,$$

or

$$p' + \frac{q^2}{32\pi^2\varepsilon_0 R^4} = p.$$

From Boyles' law, we have

$$p' = p\left(\frac{V_0}{V}\right) = p\left(\frac{4\pi R_0^3/3}{4\pi R^3/3}\right) = p\frac{R_0^3}{R^3}.$$

So we have the relation

$$p\frac{R_0^3}{R^3} + \frac{q^2}{32\pi^2\varepsilon_0 R^4} = p,$$

and

$$q^2 = 32\pi^2\varepsilon_0 pR\left(R^3 - R_0^3\right) .$$